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Amedeo Piolatto
Barcelona Institute of Economics (IEB)

Matthew D. Rablen
Brunel University

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Amedeo Piolatto*
Barcelona Institute of Economics (IEB)

Matthew D. Rablen†
Brunel University

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Abstract

The standard expected utility model of tax evasion predicts that evasion is decreasing in the marginal tax rate (the Yitzhaki puzzle). The existing literature disagrees on whether prospect theory overturns the *puzzle*. We disentangle four distinct elements of prospect theory and find loss aversion and probability weighting to be redundant in respect of the *puzzle*. Prospect theory fails to reverse the *puzzle* for various classes of endogenous specification of the reference level. These classes include, as special cases, the most common specifications in the literature. New specifications of the reference level are needed, we conclude.

JEL Classification: H26; D81; K42

Keywords: prospect theory, tax evasion, Yitzhaki puzzle, stigma, diminishing sensitivity, reference dependence, endogenous audit probability, endogenous reference level.

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*Barcelona Institute of Economics, c/ Tinent Coronel Valenzuela, 1-11, 08034 Barcelona, Spain. E-mail: piolatto[at]ub.edu

†Corresponding author: Department of Economics and Finance, Brunel University, Uxbridge, UB8 3PH, UK. E-mail: matthew.rablen[at]brunel.ac.uk

1 Introduction

If fines are imposed on the evaded tax, and if taxpayers' preferences satisfy the (theoretically and empirically plausible) assumption of decreasing absolute risk aversion (DARA), then the Expected Utility Theory (EUT) model of tax evasion predicts a negative relationship between tax rates and evasion (Yitzhaki, 1974).[1] Much empirical and experimental evidence, however, finds a positive relationship between evasion and the tax rate (see, e.g., Bernasconi *et al.*, in press, and the references therein).[2] Owing to its lack of empirical support, and its counter-intuitive nature, the negative relationship between tax rates and evasion predicted by the EUT model has sometimes been termed the "Yitzhaki paradox" or "Yitzhaki puzzle".

Prospect Theory (PT) has become standard in behavioural economics, for it is able to resolve many puzzles associated with EUT and provides a better fit to much empirical data (Bruhin *et al.*, 2010).[3] Our study seeks to (re)-examine the role of PT in reversing the Yitzhaki puzzle. In recent years a number of papers have claimed that applying the insights of PT to the tax evasion problem solves the Yitzhaki puzzle. Dhami and al-Nowaihi (2007: 171) claim to "...show that prospect theory provides a much more satisfactory account of tax evasion including an explanation of the Yitzhaki puzzle." Similar sentiments are also found in Trotin (2012), Bernasconi and Zanardi (2004) and Yaniv (1999). In their recent review of this literature, however, Hashimzade *et al.* (in press: 16) conclude (on the basis of several examples) that "Prospect theory does not necessarily reverse the direction of the tax effect: our examples show that certain choices of the reference level can affect the direction of the tax effect in some situations, but none of the examples is compelling." We investigate this dichotomy.

We analyse the tax evasion decision with variants of a Reference-Dependent (RD) model (which includes PT as a special case) in which we vary (i) the elements of PT that are assumed to hold; (ii) the properties of the reference level, which may (or may not) depend on the marginal tax rate and/or on the taxpayer's income declaration; and (iii) the properties of the probability of audit, which we allow to be fixed exogenously or to be a function of the taxpayer's declaration. In particular, we decompose PT into four distinct elements. The first, reference dependence, assumes outcomes to be judged relative to a reference level of

[1] For expositions of the EUT model, see Allingham and Sandmo (1972) and Srinivasan (1973).

[2] The empirical evidence is not entirely consistent, however. See Feinstein (1991) for a contrasting finding.

[3] PT was initially proposed by Kahneman and Tversky (1979), and subsequently extended to "cumulative" PT by Tversky and Kahneman (1992). In this study we use cumulative PT, but our qualitative conclusions apply equally to the original version of PT. See, e.g., Barberis (2013) and Camerer (2000) for reviews of further applications of PT beyond that to tax evasion.

wealth R.[4] The second, diminishing sensitivity, assumes that marginal utility is diminishing in distance from the reference level, which implies concave preferences over outcomes above the reference level and convex preferences over outcomes below the reference level. The third, loss aversion, is the property that the disutility of a loss exceeds the utility of a gain of equal magnitude. The final element, probability weighting, transforms objective probabilities into decision weights. Decomposing PT in this way allows us, unlike the existing literature, to clarify the elements which allow it to overturn the Yitzhaki puzzle or otherwise.

Our results in some cases extend, and in others contrast, with the existing literature. When both the audit probability and the reference level are treated as exogenously fixed, we find that the introduction of reference dependence does not, on its own, reverse the Yitzhaki puzzle. The combination of reference dependence and diminishing sensitivity, however, unambiguously reverses the Yitzhaki puzzle when, at the interior maximum, the payoff if caught lies below the reference level. Throughout the analysis, loss aversion and probability weighting are found to play no role in determining the ability of the RD model to overturn the puzzle.[5]

Allowing the reference level to be a decreasing function of the tax rate has curious effects. If the reference level is sufficiently *sensitive* to the tax rate, then simply the assumption of reference dependence is sufficient to reverse Yitzhaki's puzzle. On the other hand, if reference dependence and diminishing sensitivity are assumed, Yitzhaki's puzzle is reversed only if the reference level is sufficiently *insensitive* to the tax rate. We show that there exists a set of specifications of the reference level that are insufficiently sensitive to the tax rate for reference dependence alone to reverse the Yitzhaki puzzle, but that are too sensitive to the tax rate for reference dependence combined with diminishing sensitivity to reverse the Yitzhaki puzzle. Importantly, the specification of the reference level as the taxpayer's post-tax wealth if they do not evade – which is argued as the most plausible specification of the reference level by several authors – belongs to this set. In these cases, whether utility is assumed to be globally concave, or to display diminishing sensitivity, the RD model cannot reverse the Yitzhaki puzzle. These results are shown to be robust to a class of specifications of the reference level (which includes, for instance, the expected value of the tax gamble) that also allow for a dependency on the taxpayer's declaration.

[4] In their development of PT, Kahneman and Tversky (1979) assume the reference level R to be an exogenous parameter, although in many economic applications it is assumed endogenous to the parameters of the model. We consider alternative assumptions for the setting of the reference level, therefore.

[5] Consistent with this finding, Eide (2001) shows that introducing (rank-dependent) probability weighting into the standard tax evasion model changes none of the qualitative comparative statics results.

When the audit probability is made endogenous to the model, the analysis becomes more complex, and predictions less clear. When, however, we restrict our attention to the class of models in which utility is homogeneous – the case considered by Dhami and al-Nowaihi (2007) – we find that both the EUT and RD models yield clear predictions. In particular, we obtain that the RD model cannot overturn the Yitzhaki puzzle. We square this finding with that in Dhami and al-Nowaihi (2007) – who argue that PT unambiguously reverses the Yitzhaki puzzle – by noting that these authors augment the PT model with an assumed "stigma" cost associated with being caught cheating. When stigma is set to zero in their model, PT no longer overturns the Yitzhaki puzzle either. We also prove that, when the EUT model is augmented with stigma similarly, it too can overturn the Yitzhaki puzzle. We therefore find no evidence that the ability of the RD model to overturn the puzzle exceeds that of the EUT model in this case.

We do not claim that either the EUT or RD models we consider are either descriptively superior or inferior to the PT model over the full gamete of empirical regularities on tax related behaviour, and other evidence relating to behaviour in risky settings more generally. We do claim, however, that existing approaches to the application of PT to tax evasion largely fall short in respect of one of the most significant such empirical regularities: that tax evasion is increasing in the marginal tax rate. We conclude that new approaches are needed, particularly regarding the specification of the reference level.

The plan of the paper is the following. Section 2 introduces the baseline, EUT model, from which we depart in (section 3), in order to analyse the RD model under varying assumptions regarding the specification of the reference level, audit probabilities, and the utility function. Section 4 concludes. All proofs are in the Appendix.

2 The EUT model

As a springboard for our later analysis, we begin with a development of the standard EUT model. Consider a taxpayer with an exogenous taxable income Y (which is known by the taxpayer but not by the tax authority). The government levies a proportional income tax at marginal rate t on declared income X. The probability of audit is given by $p \in (0, 1)$. Following Yitzhaki (1974), audited taxpayers face a fine at rate $f > 1$ on all undeclared tax. The taxpayer's expected utility may be written as

$$V = pv\left(Y^c\right) + [1 - p]\, v\left(Y^n\right), \tag{1}$$

where $Y^n = Y - tX$ is the taxpayer's wealth when caught (audited), $Y^c = Y^n - tf[Y - X]$ is the taxpayer's wealth when not caught, and v is an increasing and strictly concave utility function. The first and second order conditions for a maximum are given by

$$\frac{\partial V}{\partial X} = t[p[f-1]v'(Y^c) - [1-p]v'(Y^n)] = 0; \tag{2}$$

$$\frac{\partial^2 V}{\partial X^2} = D = t^2\left[p[f-1]^2 v''(Y^c) + [1-p]v''(Y^n)\right] < 0; \tag{3}$$

where the latter is satisfied by the strict concavity of v. The derivative $\partial X/\partial t$, found implicitly from (2), is

$$\frac{\partial X}{\partial t} = t\frac{[1-p]Xv''(Y^n) - p[f-1][X + [Y-X]f]v''(Y^c)}{-D}. \tag{4}$$

A mode of derivation that shall prove insightful once we move to analysing variants of the RD model is to add and subtract $t^{-1}D[Y-X]$ in the numerator of (4), in which case we obtain

$$\frac{\partial X}{\partial t} = \frac{1}{t}[Y-X] - \frac{tY[p[f-1]v''(Y^c) - [1-p]v''(Y^n)]}{-D}, \tag{5}$$

to which application of (2) yields

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{Y[A(Y^n) - A(Y^c)]}{[f-1]A(Y^c) + A(Y^n)}\right], \tag{6}$$

where $A(x) = -v''(x)/v'(x)$ is the Arrow-Pratt coefficient of absolute risk aversion. If, as is conventional, we assume decreasing absolute risk aversion (DARA), i.e., $A'(x) < 0$ – which implies that $A(Y^c) > A(Y^n)$ – equation (6) yields the Yitzhaki's (1974) puzzle: *under EUT and DARA, $\partial X/\partial t > 0$ at an interior maximum.* Yitzhaki's puzzle should be understood as a pure wealth effect. A rise in the tax rate lowers expected wealth, which, under DARA, makes taxpayers more risk averse. Hence taxpayers find it optimal to evade less.

3 Departures from the EUT model

We now depart from the EUT model. We introduce variants of what we term the "reference-dependent" model that each share reference-dependence as a common assumption, but that allow additionally for further elements of PT.

Reference Dependence

Reference dependence can be introduced into the EUT model independently of the remaining elements of PT. This is performed by writing the taxpayers' objective function in (1) as:[6]

$$V_R = pv\left(Y^c - R\right) + [1-p]\, v\left(Y^n - R\right). \tag{7}$$

Diminishing sensitivity

Diminishing sensitivity cannot meaningfully be introduced into the EUT model independently of reference dependence. In equation (7) it requires utility to be convex when its argument is negative. For $x < 0$, we therefore replace $v(x)$ with $\underline{v}(x)$, where $\underline{v}'' > 0$ such that the coefficient of absolute risk aversion is $\underline{A}(x) < 0$. As is widely noted in the literature, under diminishing sensitivity an interior maximum must satisfy $Y^n - R > 0$, for otherwise the taxpayer's objective function is globally convex.[7] Moreover, if $Y^c - R > 0$,then the results with or without diminishing sensitivity are unchanged. Hence, when examining the RD model with diminishing sensitivity, we focus on the only interesting case, in which $Y^n > R > Y^c$. In this case we can write the taxpayers' objective function in as

$$V_{DS} = p\underline{v}\left(Y^c - R\right) + [1-p]\, v\left(Y^n - R\right). \tag{8}$$

The first and second derivatives of (8) with respect to X are given by

$$\frac{\partial V_{DS}}{\partial X} = t\left[p\,[f-1]\, \underline{v}'\left(Y^c - R\right) - [1-p]\, v'\left(Y^n - R\right)\right] \tag{9}$$

$$D_{DS} = \frac{\partial^2 V_{DS}}{\partial X^2} = -t^2\,[1-p]\, v'\left(Y^n - R\right)\left[A\left(Y^n - R\right) + [f-1]\, \underline{A}\left(Y^c - R\right)\right], \tag{10}$$

The second derivative in (10) is ambiguous in sign. The condition for it to be negative is that $A\left(Y^n - R\right) > -[f-1]\, \underline{A}\left(Y^c - R\right)$, which cannot be guaranteed by any easily interpretable restriction on the parameters of the model. The second order condition $D_{DS} < 0$ may, therefore, not be satisfied. Moreover, under diminishing sensitivity it is possible – because of the possibility of corner solutions – that the first and second order conditions do not describe the solution of the maximisation problem. Local maxima may also arise, so the first order condition may not possess a unique solution.[8]

[6] For axiomatisations of frameworks that allow for reference dependence separately of the remaining elements of PT see, e.g., Sugden (2003) and Apesteguia and Ballester (2009).

[7] We do not investigate the properties of corner solutions of the EUT and RD model, for the descriptive validity of tax evasion as an all-or-nothing activity appears weak. Note, in particular, that the focus on reference levels that are consistent with interior maxima rules out the specification of the reference level as $R = Y$, although this specification is allowable once we endogenise p in section 3.3.

[8] See Hashimzade *et al.* for a detailed discussion of these difficulties.

Henceforth, when analysing the RD model with diminishing sensitivity, we proceed under the maintained assumption that indeed the first order condition describes a unique, and genuinely optimal, interior choice for the taxpayer. Under this assumption equations (9) and (10) together imply that the interior maximum satisfies $[f-1]\,\underline{A}(Y^c-R)+A(Y^n-R)>0$.

Loss aversion

Loss aversion with respect to a utility function v requires that $-v(-x)>v(x)$ for $x>0$. Note that this condition necessarily holds if v is strictly concave, hence loss aversion is already implied by the EUT model and the RD model with globally concave utility.[9] Loss aversion is no longer guaranteed, however, once reference dependence and diminishing sensitivity are assumed. Under these assumptions, loss aversion holds if $\underline{v}(\cdot)$ is assumed to satisfy $-\underline{v}(-x)>v(x)$ for $x>0$.

Probability weighting

Probability weighting can be introduced in the EUT model on its own, or in combination with any of the remaining elements of PT. It may be introduced into either of equations (7) or (8) by replacing p with $w(p)$, where $w(0)=0$, $w(1)=1$ and $w'>0$.[10]

3.1 Exogenous Reference Dependence

We assume initially that R is independent of X and t. To analyse the introduction of reference dependence on its own we repeat the steps of Section 2 for equation (7) to obtain

$$\frac{\partial X}{\partial t}=\frac{1}{t}\left[[Y-X]-\frac{Y\,[A(Y^n-R)-A(Y^c-R)]}{[f-1]\,A(Y^c-R)+A(Y^n-R)}\right]. \tag{11}$$

To introduce diminishing sensitivity, we proceed identically, but using equation (8), to obtain

$$\frac{\partial X}{\partial t}=\frac{1}{t}\left[[Y-X]-\frac{Y\,[A(Y^n-R)-\underline{A}(Y^c-R)]}{[f-1]\,\underline{A}(Y^c-R)+A(Y^n-R)}\right]. \tag{12}$$

We then have our first Proposition:

[9] We use the original definition of loss aversion in Kahneman and Tversky (1979). Unlike this "global" condition, Köbberling and Wakker (2005) propose an alternative "local" definition of loss aversion – which is not satisfied by the EUT model or the RD model with concave utility – according to which v displays loss aversion if and only if $\lim_{x\uparrow 0}\partial v(x)/\partial x>\lim_{x\downarrow 0}\partial v(x)/\partial x$.

[10] Hence, the objective probability distribution is $(p,1-p)$ and the transformed probability distribution is $(w(p),1-w(p))$. PT allows for different weighting functions to apply to outcomes that fall above or below the reference level. As pointed out by Dhami and al-Nowaihi (2007) and Prelec (1998), however, empirically the same weighting function is found to apply above and below the reference level, so we assume there to be a single weighting function w.

Proposition 1 *Assume exogenous reference dependence. Then:*
(i) assuming DARA, at an interior maximum, $\partial X/\partial t > 0$.
(ii) assuming diminishing sensitivity, at an interior maximum satisfying $Y^n > R > Y^c$, $\partial X/\partial t < 0$.
(iii) parts (i) and (ii) hold if loss aversion and/or probability weighting are additionally assumed.

Part (i) of Proposition 1 makes clear that, on its own, adding reference dependence to the EUT model does not resolve the Yitzhaki puzzle. Part (ii), however, shows that combining diminishing sensitivity with exogenous reference dependence *is* sufficient to resolve the Yitzhaki puzzle. The key to the result with diminishing sensitivity is that, as expected wealth falls, taxpayers becomes *more* willing to bear risk to the extent that outcomes fall below the reference level. Bernasconi and Zanardi (2004) prove a version of part (ii), but using the full apparatus of PT. Our results in parts (ii) and (iii) make clear that only reference dependence and diminishing sensitivity are required to reverse the puzzle, however.

3.2 Endogenous Reference Dependence

We now repeat the analysis of Section 3.1 when the reference level is permitted to depend endogenously on one or more of X and t. In section 3.2.1 we consider the case when the reference level depends on the tax rate t, while in section 3.2.2 we allow it to also depend on the declared income X.

3.2.1 Reference as a function of t

We now consider the case in which $R_t = \partial R/\partial t < 0$. This is a generalisation of the case, first proposed by Dhami and al-Nowaihi (2007), and employed subsequently in the literature, in which the reference level is the taxpayer's post-tax wealth if they do not evade (the legal post-tax wealth): $R = Y[1-t]$. Assuming reference dependence only, the derivative $\partial X/\partial t$ (equation 6) becomes

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{[Y+R_t]\left[A\left(Y^n-R\right)-A\left(Y^c-R\right)\right]}{[f-1]\,A\left(Y^c-R\right)+A\left(Y^n-R\right)}\right]. \tag{13}$$

Reference dependence combined with diminishing sensitivity yields

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{[Y+R_t]\left[A\left(Y^n-R\right)-\underline{A}\left(Y^c-R\right)\right]}{[f-1]\,\underline{A}\left(Y^c-R\right)+A\left(Y^n-R\right)}\right]. \tag{14}$$

Proposition 2 *Assume $R_t < 0$ and $R_X = 0$. Then:*
(i) assuming DARA, there exists a threshold level $\widetilde{R}_t < -Y$ such that, at an interior maximum, $\partial X/\partial t < 0$ for $R_t < \widetilde{R}_t$ and $\partial X/\partial t \geq 0$ for $R_t \geq \widetilde{R}_t$.
(ii) assuming diminishing sensitivity, there exists a threshold level $\widetilde{R}_{t,DS} > -Y$ such that, at an interior maximum, $\partial X/\partial t < 0$ for $R_t > \widetilde{R}_{t,DS} > -Y$ and $\partial X/\partial t \geq 0$ for $R_t \leq \widetilde{R}_{t,DS}$.
(iii) parts (i) and (ii) hold if loss aversion and/or probability weighting are additionally assumed.

Part (i) of Proposition 2 makes clear that the right side of (13) is *increasing* in R_t. Hence, if reference level is sufficiently *sensitive* to the tax rate (R_t sufficiently negative), this effect can fully offset the wealth effect that underlies the Yitzhaki puzzle. The intuition is straightforward: once the reference level is a decreasing function of the tax rate, taxpayers need not feel poorer after an increase in the tax rate, for the fall in Y^c and Y^n is offset in the utility function by a fall in R. Indeed, if R responds more to the tax rate than does the expected value of the tax gamble, taxpayers feel richer (relative to the reference level) and the logic of Yitzhaki's finding is reversed: richer taxpayers, under DARA, become less risk averse.

Part (ii) of Proposition 2 makes clear, however, that the right side of (14) is *decreasing* in R_t. Note that, unlike part (i), this observation does not rely on the derivatives of $A(\cdot)$, for simply the difference in signs between $A(\cdot)$ and $\underline{A}(\cdot)$ is sufficient to sign $A(\cdot) - \underline{A}(\cdot) > 0$. The endogenous movement of the reference level therefore works against the effects of diminishing sensitivity. In order to ensure that the effect due to diminishing sensitivity dominates the reverse effect from the endogeneity of the reference level, the reference level must be sufficiently *insensitive* to the tax rate (R_t sufficiently close to zero). The intuition is similar to that developed for part (i), but in reverse: if R is too sensitive to the tax rate an increase in t makes taxpayers feel richer relative to the reference level, which, under diminishing sensitivity, makes them more risk averse, and evade less. If, however, R is not so sensitive to the tax rate that taxpayers feel richer following an increase in t, then the intuition developed in part (ii) of Proposition 1 continues to hold, and the Yitzhaki puzzle is reversed. The final part of Proposition 2 clarifies that, again, allowing for loss aversion and probability weighting does not alter the conclusions of the analysis.

We may now state a straightforward corollary of Proposition 2:

Corollary 1 *(i) Assume $R_X = 0$, and $R_t \in \left(\widetilde{R}_t, \widetilde{R}_{t,DS}\right)$. Then, at an interior maximum, $\partial X/\partial t > 0$ whether or not diminishing sensitivity is assumed.*

(ii) Assume $R = Y[1 - t]$, which implies $R_t = -Y$. Then, from equations (13) and (14), $\partial X/\partial t = t^{-1}[Y - X] > 0$ whether or not diminishing sensitivity is assumed.

According to part (i) of Corollary 1, there is an interval of R_t such that the reference level is insufficiently sensitive to the tax rate for the RD model without diminishing sensitivity to resolve the Yitzhaki puzzle, but too sensitive for the RD model with diminishing sensitivity to resolve the Yitzhaki puzzle. Hence, in this range, whatever the form of v, the RD model cannot resolve the Yitzhaki puzzle. Note that, in this case, the ability of the RD to reverse the Yitzhaki puzzle is strictly weaker than that of the EUT model. The latter can always reverse the puzzle, albeit by invoking the unsatisfactory assumption of increasing absolute risk aversion (and this must be sufficiently strong), whereas the RD model cannot reverse the puzzle for any choice of preferences consistent with an interior maximum. Part (ii) clarifies, for emphasis, that the result in part (i) applies to the specification of R as the legal post-tax wealth – as adopted in much of the literature.

The preceding findings have implications for some of the existing literature. First, Yaniv (1999) claims to resolve the Yitzhaki puzzle in a PT model with the reference level specified as $R = Y - D$, where D is the amount of an advance tax payment. The advance payment D is specified (up to a constant) as $D = \alpha t b$, implying $R_t = -\alpha b$, where b is the tax authority's estimate of the taxpayer's income (which could under- or over-estimate the true Y), and $\alpha \in [0, 1]$. By Proposition 2, a necessary (and still not sufficient) condition for $\partial X/\partial t < 0$ in this context is that $R_t > -Y$, which implies $\alpha b < Y$. For this condition to hold for any $\alpha \in [0, 1]$ it must be that $b < Y$. The result claimed by Yaniv is therefore subject to potentially important qualifications. Second, in an unpublished working paper, Trotin (2012) claims (her Proposition 8) to resolve the Yitzhaki puzzle in a PT model with the reference level as the taxpayer's legal post-tax wealth. Corollary 1 shows this proposition to be false.[11]

3.2.2 Reference as a function of t and X

We now turn to the case in which the reference level is an endogenous function of both the marginal tax rate and the taxpayer's declaration. Although we know of no detailed application to tax evasion that employs a reference level of this form, it is of interest for at least two reasons. First, Kőszegi and Rabin (2006) make a general argument, designed to be

[11] The difference in findings appears due to a error in the proof of Trotin's Proposition 8. In particular, we are unable to replicate the expression for $\partial \Phi_{\overline{R}}(x^*, t)/\partial t$ in the first line of the proof.

portable across contexts, that the reference level should reflect the expected outcome of the lottery. If, accordingly, the reference level is set as the expected value of the tax gamble,

$$R = pY^c + [1-p]\,Y^n = Y\,[1-pft] + tX\,[pf-1]\,, \tag{15}$$

then it is a function of both X and t.[12] Second, Hashimzade *et al.* (in press) briefly consider an example in which $R = [1-t]\,X$, but do not draw out the more general implications of allowing for dependency upon X. Note that each of the aforementioned reference levels satisfies the properties that, at an interior maximum, $R_t < 0$, $R_X < 0$, $R_{XX} = 0$ and R_X is homogeneous of degree one in t, such that $t^{-1}R_X$ is independent of t.[13] If, accordingly, we endow R with these properties then a change in the tax rate affects the declared income in the following way:[14]

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{\phi\,[A\,(Y^n-R) - A\,(Y^c-R)]}{[ft-t-R_X]\,A\,(Y^c-R) + [t+R_X]\,A\,(Y^n-R)}\right], \tag{16}$$

or, for the case of diminishing sensitivity,

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[[Y-X] - \frac{\phi\,[A\,(Y^n-R) - \underline{A}\,(Y^c-R)]}{[ft-t-R_X]\,\underline{A}\,(Y^c-R) + [t+R_X]\,A\,(Y^n-R)}\right], \tag{17}$$

where $\phi = t\,[Y+R_t] + R_X\,[Y-X]$.

Proposition 3 *Assume $R_t < 0$, $R_X < 0$, $R_{XX} = 0$ and R_X homogeneous of degree one in t. Then parts (i)-(iii) of Proposition 2 hold unchanged, and so does its Corollary 1.*

Proposition 3 is a strong result: it states that additionally allowing the reference level to depend upon X (as well as t) in the manner so far considered in the literature leaves the predictive power of the RD model in respect of the Yitzhaki puzzle entirely unaltered. The

[12] This specification for R guarantees that, for any $X \in [0,Y]$, the taxpayer's wealth is (weakly) below the reference level if caught ($-ft\,[1-p]\,[Y-X] \geq 0$) and (weakly) above the reference level if caught ($pft\,[Y-X] \geq 0$) for any $X \in [0,Y]$. This property is also a feature of the specification of the reference level as the legal post-tax wealth that we analysed in section 3.2.1. Some of the previous literature claims this property of the reference level to be unique to the legal post-tax wealth (e.g., Proposition 3 in Dhami and al-Nowaihi, 2007). Our counterexample proves this claim incorrect: any convex combination of wealth when caught and when not caught possesses this property.

[13] To sign R_X in the case in which R is the expected value of the tax gamble, we make use of the fact that $pf < 1$ at an interior maximum. This is the standard condition that the tax gamble must be better than fair.

[14] We write the FOC as $\partial V/\partial X = t\left[p\left[[f-1] - t^{-1}R_X\right]v'\,(Y^c) - [1-p]\left[1+t^{-1}R_X\right]v'\,(Y^n)\right] = 0$. As $t^{-1}R_X$ is independent of t, we may then simply apply the steps of Section 2.

proof proceeds by establishing that equations (16) and (17) have identical roots to (13) and (14). Hence, it remains the case that, for any reference level such that $R_t \in \left(\widetilde{R}_t, \widetilde{R}_{t,DS}\right)$, the RD model is unable to reverse Yitzhaki's puzzle whether or not diminishing sensitivity is assumed. Is this finding germane to the specification of the reference level as the expected value of the tax gamble, or as $R = [1-t]\,X$?

Corollary 2 *If R is the expected value of the gamble, or if $R = [1-t]\,X$ as in Hashimzade et al. (in press), then $\partial X/\partial t > 0$ whether or not diminishing sensitivity is assumed.*

According to Corollary 2 neither of these specifications of the reference level can overturn the Yitzhaki puzzle in any variant of the RD model. Together, the results of sections 3.2.1 and 3.2.2 imply that the RD model does not reverse the Yitzhaki puzzle for any of the endogenous specifications of the reference level we observe in the literature.

3.3 Endogenous Audit Probability

Suppose now that the probability of audit is not exogenous, but instead depends on declared income.[15] Consistent with the literature on optimal auditing (e.g., Reinganum and Wilde, 1986) we assume that higher income declarations are less likely to be audited ($\partial p/\partial X \leq 0$). The models discussed so far are for the special case of this assumption in which $\partial p/\partial X = 0$. Under this new assumption the analysis becomes more complex and few, if any, general results are possible. We therefore follow Dhami and al-Nowaihi (2007) who, citing Tversky and Kahneman (1992), analyse a model in which v is homogeneous of degree $\beta \in [0,1]$.[16] In this framework, equation (1) becomes

$$V_p = pv\left(Y^c - R\right) + [1-p]\,v\left(Y^n - R\right), \tag{18}$$

where now $p = p\,(X)$. By homogeneity, equation (18) becomes

$$V_p = [Y-X]^{\beta} t^{\beta} \left[p\left[[1-f]^{\beta} + 1 - p\right] v(1)\right]. \tag{19}$$

The first order condition corresponding to (19) is

$$[Y-X]^{\beta} t^{\beta} \left[-\left[p\,[1-f]^{\beta} + 1 - p\right] v'(1) + \left[[1-f]^{\beta} - 1\right] p' v(1)\right] = 0. \tag{20}$$

The next proposition characterises how the introduction of an endogenous audit probability alter the predictions of the RD model, under the assumption of homogeneity.

[15] Hashimzade *et al.* (in press) discuss this version of the RD model only cursorily in their footnote 5.

[16] The homogeneous form is standard in applications of PT, and is axiomatised under PT by al-Nowaihi *et al.* (2008).

Proposition 4 *Assume endogenous reference dependence, with $R = Y[1-t]$, v homogeneous of degree β, and $p' \leq 0$. Then, at an interior maximum, $\partial X/\partial t = 0$.*

Proposition 4 clarifies that, when the reference level is the legal post-tax level of wealth, the RD model with homogenous preferences makes the same prediction for the relationship between the tax rate and evasion at an interior maximum, irrespective of whether $p'(X) = 0$ or $p'(X) < 0$ is assumed. In either case, Yitzhaki's puzzle remains. Moreover, Proposition 4 holds irrespective of whether utility is assumed globally concave, or to display diminishing sensitivity. The only distinction of note between the two cases is that, as noted by Dhami and al-Nowaihi (2007), for $p'(X) = 0$ the dynamics of the optimum are bang-bang. Hence, except in the special case in which an interior solution is weakly optimal, the RD model is simply incapable of delivering an interior solution for X. This difficulty is, however, mitigated when $p'(X) < 0$, for the function $p(X)$ can be chosen to make the taxpayer's objective function strictly concave.

How can Proposition 4 be squared with Proposition 4 of Dhami and al-Nowaihi (2007), which these authors interpret as showing that PT resolves the Yitzhaki puzzle? The answer is that these authors allow for a feature additional to those of PT: a "stigma" cost $s > 0$, such that wealth when caught becomes $Y_s^c = Y^c - s[Y - X]$.[17] Rewriting in our notation the expression for $\partial X/\partial t$ in their equation (8.26), we obtain

$$\frac{\partial X}{\partial t} = -s\frac{\beta\theta[s + [f-1]\,t]^{\beta-1}}{t[Y-X]^{1-\beta}}\left[\beta w(p) - [Y-X]\,w'(p)\,p'\right], \tag{21}$$

where θ is a parameter such that $\theta > 1$ implies loss aversion in their formulation. For $s > 0$, and assuming $p' < 0$, equation (21) indeed yields $\partial X/\partial t < 0$, and this result continues to hold without the assumptions of loss aversion and probability weighting ($w(p) = p$, $\theta = 1$). When stigma is removed from the model ($s = 0$), however, equation (21) yields $\partial X/\partial t = 0$, which accords with our Proposition 4: the Yitzhaki puzzle remains. Our next proposition shows that the inclusion of stigma enables the reversal of the Yitzhaki puzzle also.

Proposition 5 *Assume EUT, stigma, $p' < 0$, and risk neutrality. Then, at an interior maximum, $\partial X/\partial t < 0$.*

Proposition 5 clarifies that, once stigma is introduced into the EUT model, it too may readily reverse the Yitzhaki puzzle. Although we believe the Proposition as stated to be new,

[17] See Dhami and al-Nowaihi (2007) for a discussion on the empirical relevance of stigma and its use in the tax evasion literature.

the idea that stigma can overturn the Yitzhaki puzzle in the EUT model is not. Variations of this idea, but under different assumptions over how stigma enters the taxpayer's objective function are found in, e.g., al-Nowaihi and Pyle (2000), Dell'Anno (2009), Gordon (1989) and Kim (2003).

Proposition 5 appears of roughly equal generality to Dhami and al-Nowaihi's Proposition 4. In particular, the latter proposition need no longer hold for sufficient deviations from the assumption of homogeneity, while the former need no longer hold for sufficient deviations from risk neutrality. Overall, therefore, we find no evidence to suggest that the RD model systematically improves upon the predictions of the EUT model in respect of the Yitzhaki puzzle in this case.

Although any positive level of stigma is sufficient to overturn the Yitzhaki puzzle in the EUT model of Proposition 5, much larger levels of stigma must be assumed to resolve a further difficulty with the EUT model: it predicts far more tax evasion than is empirically observed.[18] By contrast – as loss aversion and probability weighting help reduce predicted evasion levels – PT is shown by Dhami and al-Nowaihi (2007) to be able to match empirically observed levels of evasion for much more moderate levels of the parameter s. Thus, it can be argued, the PT model should be preferred to the EUT model on these grounds.[19] We recognise this argument, but note two points. First, its validity or otherwise is orthogonal to our analysis, which is concerned solely with the ability of models to resolve the Yitzhaki puzzle. Second, it is equally possible to resolve the levels puzzle without resort to either PT or stigma costs. For instance, PT assumes that taxpayers observe the true audit probability p, which is then psychologically exaggerated (for small p) in the decision-making process. An alternative view is that taxpayers face ambiguity over p, the value of which they do not know for sure. Snow and Warren (2005) show that introducing ambiguity over p into the tax evasion model decreases predicted evasion if taxpayers are ambiguity averse. Also, Kleven *et al.* (2011) show that when the EUT model is extended to allow for plausible levels of third-party reporting, the predicted level of compliance falls to levels in line with those observed empirically. The latter explanation can be straightforwardly integrated into the EUT and RD models we consider here, so as to make them consistent with level data, without altering the predictions of these models concerning the Yitzhaki puzzle.

[18] See, e.g., Alm *et al.* (1992: footnote 3) for a detailed discussion of the levels puzzle, and al-Nowaihi and Pyle (2000) for the levels of stigma needed to resolve it.

[19] We are grateful to Sanjit Dhami for this point.

4 Conclusion

Albeit with limitations, (see, e.g., Levy and Levy, 2002; List, 2003), PT is widely viewed as the best available description of how people behave in risky settings. Barberis (2013: 73) notes, however, that there are "few well-known and broadly accepted applications of prospect theory in economics." The reason, Barberis argues, is that PT is not straightforward to apply: in particular, the most appropriate choice of the reference level is often unclear.[20]

In this paper we focused on tax evasion and in particular on the Yitzhaki puzzle: the EUT model of tax evasion predicts a decrease in tax evasion when the tax rate increases. We do this by disentangling the different elements of PT, so as to obtain a clear understanding of the forces needed to reverse the puzzle. We proceed with a general formulation that encompasses the most common specifications found in the literature. We agree with Bernasconi and Zanardi (2004) that the PT model with an exogenous reference level can reverse the Yitzhaki puzzle in some well-defined situations. As such, Hashimzade *et al.* are wrong to dismiss a role for non-expected utility theory (and PT, in particular) in reversing the Yitzhaki puzzle. Equally, however, we concur with these authors that PT, at least as so far applied in the literature, does not unambiguously improve upon the descriptive validity of the EUT model in respect of the Yitzhaki puzzle.

Barberis's generic point over the difficulty of proper identification of the reference level shines through in the tax evasion context. In particular, when the reference level is exogenous, the PT assumption of diminishing sensitivity enables it to decisively reverse the Yitzhaki puzzle. But, when the reference level is a decreasing function of the tax rate – as is the case for all psychologically plausible specifications of the reference level advanced so far in the literature – PT typically ceases to reverse the Yitzhaki puzzle. In this sense, different views over the interpretation of the reference level yield (very) different perspectives on the plausibility of PT as an explanation of the Yitzhaki puzzle.

What do our findings suggest for the importance of the individual elements of PT? We show in Propositions 1 and 2 that diminishing sensitivity is neither necessary nor sufficient for the RD model to reverse the Yitzhaki puzzle. It is not necessary as Yitzhaki's puzzle can be reversed by endogeneity of the reference level alone, and it is not sufficient, as it does not always reverse the puzzle. Curiously, diminishing sensitivity lies behind both the ability of PT to reverse the Yitzhaki puzzle when reference level is exogenous, and for its frequent

[20] Existing parameterisations of the value and weighting functions of PT are also problematic (see, e.g., Neilson and Stowe, 2002).

inability to do so when the reference level is a decreasing function of the tax rate.

We find that loss aversion and probability weighting are irrelevant in respect of the predictions of the RD model for the sign of $\partial X/\partial t$. Invoking Occam's razor, we believe that results relating to the Yitzhaki puzzle that have been attributed to "prospect theory" may more properly be interpreted as being attributable to simpler RD models that contain only a subset of the elements of PT.

We do not take our findings to imply that PT is unimportant for the tax evasion decision. Indeed, given the range of systematic deviations from EUT that PT can explain, we would be surprised if this were the case. Our findings do, however, suggest that existing approaches to the application of PT to tax evasion need to be reconsidered. We see two strands of research that might further illuminate the role of PT. The first is the further investigation of the specification of the reference level: are there psychologically plausible specifications of the reference level that satisfy the conditions required for PT to resolve the Yitzhaki puzzle? As the reference level must be sufficiently insensitive to the tax rate for PT to reverse the puzzle (Proposition 2) one possibility is to assume an adaptive process for R in an explicitly dynamic framework. In this vein, Bernasconi *et al.* (in press) allow for the reference level to adapt over time to changes in the tax rate and show that, under these conditions, PT can predict an upward drift in tax evasion (after an initial fall), following an increase in the tax rate.

Alternatively, it has long been known that taxpayers do not, in the most part, treat the evasion decision as a simple gamble (e.g., Baldry, 1986). Researchers might, therefore, investigate whether PT adds value in combination with other plausible developments of the standard model. For instance, Rablen (2010) introduces PT into a version of the tax evasion model that allows for taxes to fund the provision of a public good. The author shows that reference dependence and diminishing sensitivity are sufficient to overcome a puzzling result that arises under expected utility: that taxpayer evasion is decreasing (increasing) in the tax rate when the public good is overprovided (underprovided). For now, however, Yitzhaki's puzzle remains a puzzle.

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Appendix

Proof of Proposition 1. (i) Under DARA $A(Y^n - R) - A(Y^c - R) < 0$, hence $\partial X/\partial t > [Y - X]/t > 0$.

(ii) We may rearrange (12) to obtain

$$\frac{\partial X}{\partial t} = \frac{1}{t}\left[\frac{[Y - X]\, f\underline{A}\,(Y^c - R) - X\left[A\,(Y^n - R) - \underline{A}\,(Y^c - R)\right]}{[f-1]\,\underline{A}\,(Y^c - R) + A\,(Y^n - R)}\right] < 0. \tag{A.1}$$

(iii) Introducing loss aversion and/or probability weighting in equations (7) or (8) leaves (11) and (12) symbolically unchanged. ■

Proof of Proposition 2. (i) For a fixed R, the optimal X is independent of R_t. Hence, for a fixed R, the right side of (13) is monotonically decreasing and unbounded below for $R_t \to -\infty$. Therefore, as $\partial X/\partial t > 0$ at $R_t = 0$ by Proposition 1, there must exist a value $R_t = \widetilde{R}_t < 0$ such that $\partial X/\partial t = 0$. By monotonicity, it follows that $\partial X/\partial t < 0$ for all $R_t < \widetilde{R}_t$ and $\partial X/\partial t \geq 0$ for $R_t \geq \widetilde{R}_t$.
From equation (13), $\frac{\partial X}{\partial t} = 0 \Leftrightarrow \widetilde{R}_t = -Y + [Y - X]\,\frac{[f-1]A(Y^c-R)+A(Y^n-R)}{A(Y^n-R)-A(Y^c-R)} < -Y$.

(ii) Proceeding as in part (i), there must exist a value $R_t = -\widetilde{R}_{t,DS} < 0$ such that $\partial X/\partial t = 0$. By monotonicity, it follows that $\partial X/\partial t < 0$ for all $R_t > \widetilde{R}_{t,DS}$, and $\partial X/\partial t \geq 0$ for all $R_t \leq \widetilde{R}_{t,DS}$.
From equation (14), $\frac{\partial X}{\partial t} = 0 \Leftrightarrow \widetilde{R}_{t,DS} = -Y + [Y - X]\,\frac{[f-1]\underline{A}(Y^c-R)+A(Y^n-R)}{A(Y^n-R)-\underline{A}(Y^c-R)} > -Y$.

(iii) Introducing loss aversion and/or probability weighting leaves (13) and (14) symbolically unchanged. ■

Proof of Proposition 3. (i) Setting equation (16) to zero and re-arranging for R_t we obtain $\widetilde{R}_t = -Y + [Y - X]\,\frac{[f-1]A(Y^c-R)+A(Y^n-R)}{A(Y^n-R)-A(Y^c-R)}$.

(ii) Similarly, but using equation (17), we obtain $\widetilde{R}_{t,DS} = -Y + [Y - X]\,\frac{[f-1]\underline{A}(Y^c-R)+A(Y^n-R)}{A(Y^n-R)-\underline{A}(Y^c-R)}$.

(iii) Introducing loss aversion and/or probability weighting leaves (16) and (17) symbolically unchanged. ■

Proof of Corollary 2. If R is the expected value of the gamble then, from 15, we have $R_t = -pfY + X\,[pf - 1]$ and $R_X = -t\,[1 - pf]$. Hence $\phi = 0$, so $\partial X/\partial t = t^{-1}[Y - X] > 0$.
If, alternatively, $R = [1 - t]\,X$ then $R_t = -X$. We know from Proposition 3 that $\partial X/\partial t > 0$ for any $R_t > \widetilde{R}_t$, with $\widetilde{R}_t < -Y$. We have then $R_t = -X > -Y > \widetilde{R}_t$. ■

Proof of Proposition 4. Equation (20) can be rewritten as $\left[p\,[1-f]^{\beta} + 1 - p\right] v'(1) = \left[[1-f]^{\beta} - 1\right] p'v(1)$, which does not depend on t. ■

Proof of Proposition 5. The objective function under risk neutrality ($v(X) = X$) is given by $V = p[Y^c - s[Y - X]] + [1 - p][Y^n]$, from which we obtain the first order condition

$$-p'f[Y - X] + pf - 1 = \frac{[p'[Y - X] - p]s}{t}. \quad \text{(A.2)}$$

The derivative of t with respect to X is

$$\frac{\partial X}{\partial t} = -\frac{-p'f[Y - X] + pf - 1}{D}, \quad \text{(A.3)}$$

where $D = \partial^2 V/\partial X^2 < 0$. Using (A.2) into (A.3), we obtain

$$\frac{\partial X}{\partial t} = -\frac{[p'[Y - X] - p]s}{tD} = \frac{s}{[s + tf]D} < 0. \quad \text{(A.4)}$$

■