Tax evasion and debt in a dynamic general equilibrium model

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Abstract

Tax evasion is one of the most studied and the least desired effects of government intervention in the economy. Public expenditure contributes to the economic growth through the provision of infrastructure that increase the productivity of private capital: stagnation and low economic growth may arise as a consequence of tax evasion in a self fuelling mechanism. Most of the model proposed by the literature are set in a static framework, but the impact of tax evasion on growth is clearly a long term problem. In this article we propose an endogenous growth model to analyze the relation between tax evasion and public debt accumulation. We depart from the existing literature because we study under which conditions the economic growth can be maximised in the presence of debt which converges to a stable equilibrium. We show that for a log-utility evasion has no effect on the debt/GDP ratio and for this case we find the optimal level of audit and tax rate that maximises growth for any level of this ratio. For the more general case, it is possible to determine the conditions to maximise the growth but the solution is highly non-linerar.

1 Introduction

Tax evasion is one of the most studied and the least desired effects of government intervention in the economy. Since the seminal papers by Allingham and Sandmo (1972) and Yitzhaki (1974), the literature has offered several explanations and possible solutions for this phenomenon. Despite these efforts, tax evasion seems to be increasing.

The most recent estimates (Feige and Cebula, 2011) show that intentional under-reporting of income is about 18-19% of the total reported income in the US, leading to a tax gap of about 500 billion dollars. In Europe, the level of tax

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evasion is about 20% of GDP, accounting for a potential loss of about 1 trillion Euros each year (Buehn and Schneider, 2012; Murphy, 2014). Reducing tax evasion is a priority for most Governments, both in developed and developing countries since the revenue loss is only the tip of the iceberg as concerns the effect of tax evasion (Slemrod 2007; Alm 2012; Dzhumashev and Gahramanov 2011; Markellos et al. 2016). Public expenditure contributes to the economic growth through the provision of infrastructure that increase the productivity of private capital or by providing merit goods that may increase the productivity of social capital in the long run: stagnation and low economic growth may arise as a consequence of tax evasion in a self fuelling mechanism (Dzhumashev et al. (2020). For some countries the increase in tax evasion has meant an increase in public debt to avoid the necessary reduction in public expenditure. The relationship between tax evasion and public debt is not clear, in spite of a growing literature. In the short run, public debt allows to increase public expenditure without increasing the tax rate or reducing tax evasion, but it also generates more expenditure though the payment of interest on the debt. The macroeconomic literature has proposed several models to study the limits to sovereign debt using several approaches (see Fournier and Fall, 2017 for a review). In this paper we build on the approach proposed by Levaggi and Menoncin (2020) that compute the dynamics of the optimal debt/GDP ratio to assess under which conditions it converges towards a finite equilibrium value, endogenous to the model. We use a model similar to that proposed by Hori and Maebayashi (2019) who assume that public expenditure is used to finance merit good, but we depart from the standard approach to debt sustainability because we use this model to compute the dynamics of the optimal debt/GDP ratio when tax evasion is possible and to assess under which conditions this dynamics converges towards a finite equilibrium value, which is endogenous to the model. Thus, here we use an approach which is more in the Blanchard et al. (1990) spirit. In this setting we will also study the optimal level of the tax rate and other fiscal parameters that maximise growth.

The paper is organised as follows after reviewing the literature, in Section 3 we present the model; in Section 4 the results of our analysis are presented while in Section 5 we conclude.

2 Related literature

The literature has long analyzed the optimal taxation policies in economies where tax evasion is widespread, but very little has been produced on the relationship between public debt, tax evasion and long term prospects of the economy. Chen (2003) examines how tax evasion affects the optimal tax rate in a an AK endogenous growth model with productive public expenditures and a balanced-budget rule. Similarly to Barro (1990), his model reproduces the inverted U-shaped curve between optimal taxation and growth and shows that the optimal optimal tax rate is higher as tax evasion becomes more widespread. However, this result holds only when the government has no other instruments to finance public spending. Marakbi and Villieu (2018) introduce debt and study a two step model where at the first stage evasion is an exogenous fraction of the government's revenues. As a second step, they endogenize tax evasion and consider the optimizing behaviour of households who make an effort to evade as much taxes as possible. In this setting they show that several equilibria may arise in the steady state; a high-growth and low-public debt solution and a low-growth and high-public debt solution. As concerns the long run consequences of tax evasion on growth and public debt, the initial solution and the productivity of public expenditure are compatible with several solutions. The model proposed is rather peculiar in the assumptions used and does not allow to obtain results for more general cases. Our model will instead consider the problem of a consumer that wants to maximise consumptions streams in his/her life, it is fully rational and can anticipate Government choices. As in Hori and Maebayashi (2019) we assume that expenditure is used to finance the provision of merit goods, i.e. public expenditure does not contribute directly to economic growth. We show that for a log-utility evasion has no effect on the debt/GDP ratio and for this case we find the optimal level of audit and tax rate that maximises growth for any level of this ratio. For the more general case, it is possible to determine the conditions to maximise the growth but the solution is highly non-linear.

3 The model

We model a small open economy where prices and interest rate (r) are determined by market clearing conditions that do not depend on domestic variables. Consumers are endowed with an initial capital k_{t_0} which is used as sole input in a deterministic Ak production function:

$$y_t = A_t k_t, \tag{1}$$

where k_t is capital in per worker terms and A_t is technological productivity parameter.

A representative consumer maximises his/her inter-temporal utility which depends on the consumption of a private good (c_t) and a public good (g_t). Contrary to the literature on endogenous growth (Barro, 1990), we assume that public goods are consumption goods. Our assumption is justified since, starting from the inception of the welfare state, government expenditure goods such as health care, education, and other personal services has been increasing over time and in western countries represent the biggest share of public expenditure (OECD, 2019). While some of these services may have a long-run effect on social capital productivity, they are mostly consumption goods. Public good is financed through two tools: (i) a linear income tax at rate τ levied on the production, and (ii) a public deficit. The deficit is financed by issuing bonds which serves an interest rate r.

3.1 Consumer's preferences

The representative agent takes utility from consuming both a private produced good (c_t) and a public produced good (g_t). We assume that agent's preferences belong to the Constant Elasticity of Substitution (CES) family. Thus, we can use the following utility function:

$$U(c_t, g_t) = \frac{\chi}{1-\delta}\left(\alpha_c c_t^{1-\beta} + \alpha_g g_t^{1-\beta}\right)^{\frac{1-\delta}{1-\beta}}, \tag{2}$$

in which α_c and α_g are the relative preference weights of the private and public produced goods, respectively. The parameter δ measures the risk aversion (it coincides with the Arrow-Pratt risk aversion index), and the parameter β measures the inverse of the elasticity of substitution.

This function entails many particular forms that are commonly used in the economic literature:

- if $\beta = \delta$ the function becomes a linear combination of two Hyperbolic Absolute Risk Aversion utilities:

$$U(c_t, g_t) = \chi\alpha_c \frac{c_t^{1-\delta}}{1-\delta} + \chi\alpha_g \frac{g_t^{1-\delta}}{1-\delta},$$

- if $\beta = 1$ and $\alpha_c + \alpha_g = 1$ we get

$$\lim_{\beta\to 1} \frac{\chi}{1-\delta} e^{\frac{1-\delta}{1-\beta}\ln\left(\alpha_c c_t^{1-\beta} + \alpha_g g_t^{1-\beta}\right)} = \frac{\chi}{1-\delta} c_t^{\alpha_c(1-\delta)} g_t^{\alpha_g(1-\delta)},$$

  and if furthermore $\delta = 0$

$$U(c_t, g_t) = \chi c_t^{\alpha_c} g_t^{\alpha_g},$$

  which is the Cobb-Douglas utility function;

- if $\beta = 0$ we obtain a function which is a kind of CRRA function defined on the weighted sum of private and public consumption:

$$U(c_t, g_t) = \frac{\chi}{1-\delta}\left(\alpha_c c_t + \alpha_g g_t\right)^{1-\delta},$$

- if $\delta = 1$ and we assume that the utility function has the form $U(c_t, g_t) - \frac{\chi}{1-\delta}$; in this case, if we take the limit, we get

$$\lim_{\delta\to 1} \chi \frac{\left(\alpha_c c_t^{1-\beta} + \alpha_g g_t^{1-\beta}\right)^{\frac{1-\delta}{1-\beta}} - 1}{1-\delta} = \frac{1}{1-\beta}\ln\left(\alpha_c c_t^{1-\beta} + \alpha_g g_t^{1-\beta}\right),$$

  which is a log-function of the weighted sum of the two consumption. If furthermore we set $\beta \to 1$ and $\alpha_g = 1 - \alpha_c$, we get

$$\alpha_c \ln c_t + (1-\alpha_c)\ln g_t,$$

  which is a weighted sum of two log-functions.

3.2 Capital accumulation and public debt

The agent is endowed with an initial amount of capital k_{t_0}. The capital:

- increases because of production; here, we assume that the produced yield y_t is a linear function of capital

$$y_t = Ak_t, \tag{3}$$

- increases because of the interest rate on the public debt bought by the agent: $B_t r$;
- decreases because of the increment in public debt that must be financed by private capital (dB_t);
- decreases because of consumption of the private good c_t;
- decreases because of taxes; here, we assume that tax is a constant percentage (τ) of yield;
- the agent can hide a percentage e_t of his yield to the tax authority, but if he is caught evading, he must pay a fine η on the evaded income; i.e the fine will be equal to $\eta e_t y_t$.

If the event of a tax audit is represented by a jump Poisson process $(d\Pi_t)$ with constant intensity (frequency) λdt, the capital accumulation (dynamics) is given by

$$dk_t = (Ak_t - c_t - \tau(1 - e_t) Ak_t + rB_t) dt - \eta e_t Ak_t d\Pi_t - dB_t. \tag{4}$$

On the side of the government (that coincides with the tax authority), the public debt:

- increases because of the interest rate to be paid on the total debt $(B_t r)$;
- increases because of the production of the public good g_t;
- decreases because of the taxes that are received; here, we assume that the government losses a percentage (ϕ) of the taxes because of both inefficiencies and operational costs;
- increases because of the audit cost; we assume that this cost is proportional to both the yield and the square of the audit frequency $(\frac{\omega}{2}\lambda^2 y_t)$;
- when an evasion fee is paid by the agent, the public debt suddenly decreases.

Thus, we can write the debt dynamics and follows

$$dB_t = \left(B_t r + g_t - (1-\phi)\tau(1-e_t) Ak_t + \frac{\omega}{2}\lambda^2 Ak_t\right) dt - (1-\phi)\eta e_t Ak_t d\Pi_t, \tag{5}$$

in which dB_t is the public deficit. If we substitute the debt differential into the capital dynamics, we get

$$dk_t = \left(\left(1 - \frac{\omega}{2}\lambda^2 - \phi\tau\left(1 - e_t\right)\right) Ak_t - c_t - g_t\right) dt - \phi\eta e_t Ak_t d\Pi_t. \qquad (6)$$

We see that when there is no inefficiency (i.e. $\phi = 0$), the tax evasion has no effect on the capital dynamics, because any amount of capital subtracted from the production of public good, directly reduces the utility of the agent.

Remark 1. The expected increment in capital is

$$\mathbb{E}_t\left[dk_t\right] = \left(\left(1 - \frac{\omega}{2}\lambda^2 - \phi\tau + \phi\left(\tau - \eta\lambda\right) e_t\right) Ak_t - c_t - g_t\right) dt.$$

Thus, we see that if $\eta\lambda = 1$ the agent is indifferent whether to evade or not.

3.3 The optimization problem

We assume that the agent has an infinite time horizon and a constant subjective discount rate ρ. Accordingly, his optimization problem can be written as

$$\max_{\{c_t, g_t, e_t\}_{t\in[t_0,\infty[}} \mathbb{E}_{t_0}\left[\int_{t_0}^{\infty} U\left(c_t, g_t\right) e^{-\rho(t-t_0)} dt\right], \qquad (7)$$

given the capital dynamics (6).

Proposition 2. *The optimal consumption and evasion solving Problem (7), given the capital dynamics (6), are*

$$e_t^* = \frac{1}{\phi\eta\tau A}\left(1 - \left(\eta\lambda\right)^{\frac{1}{\delta}}\right), \qquad (8)$$

$$g_t^* = k_t \frac{\left(\rho + \lambda\right)\frac{1}{\delta} + \frac{\delta-1}{\delta}\left(1 - \frac{\omega}{2}\lambda^2 - \tau\phi\right) A + \frac{\delta-1}{\delta}\frac{1}{\eta} - \lambda\left(\eta\lambda\right)^{\frac{1}{\delta}-1}}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} + 1}, \qquad (9)$$

$$c_t^* = k_t \left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} \frac{\left(\rho + \lambda\right)\frac{1}{\delta} + \frac{\delta-1}{\delta}\left(1 - \frac{\omega}{2}\lambda^2 - \tau\phi\right) A + \frac{\delta-1}{\delta}\frac{1}{\eta} - \lambda\left(\eta\lambda\right)^{\frac{1}{\delta}-1}}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} + 1}. \qquad (10)$$

Proof. See Appendix A. □

In this framework the optimal evasion is a constant fraction of income, and the the optimal consumption is a constant fraction of capital.

The case without evasion is obtained when the fiscal parameters are such that (as shown in Remark 1)$\eta\lambda = 1$.In this case, the optimal evasion is zero and the optimal consumption are shown in the following corollary.

Corollary 3. *When $\eta\lambda = 1$ the optimal solutions to Problem (7) are*

$$e_t^* = 0, \tag{11}$$

$$\frac{g_t^*}{k_t} = \frac{1}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} + 1} \left(\rho\frac{1}{\delta} + \frac{\delta - 1}{\delta}\left(1 - \frac{\omega}{2}\lambda^2 - \tau\phi\right) A\right), \tag{12}$$

$$\frac{c_t^*}{k_t} = \frac{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}}}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} + 1} \left(\rho\frac{1}{\delta} + \frac{\delta - 1}{\delta}\left(1 - \frac{\omega}{2}\lambda^2 - \tau\phi\right) A\right). \tag{13}$$

We see that the following term

$$\rho\frac{1}{\delta} + \frac{\delta - 1}{\delta}\left(1 - \phi\tau\right) A,$$

with $\delta > 0$ is a weighted average of two rates: the subjective discount rate ρ, and the net of tax productivity $(1 - \phi\tau) A$.

3.4 The optimal capital accumulation

When the optimal consumption and evasion are substituted into the capital accumulation (6), we obtain

$$\frac{dk_t}{k_t} = \left(\frac{\left(1 - \frac{\omega}{2}\lambda^2 - \tau\phi\right) A - \rho + \frac{1}{\eta} - \lambda}{\delta}\right) dt - \left(1 - (\eta\lambda)^{\frac{1}{\delta}}\right) d\Pi_t.$$

The expected capital growth rate is

$$\gamma^* := \frac{1}{dt}\mathbb{E}_t\left[\frac{dk_t}{k_t}\right] = \frac{\left(1 - \frac{\omega}{2}\lambda^2 - \tau\phi\right) A - \rho + \frac{1}{\eta} - \lambda}{\delta} - \lambda\left(1 - (\eta\lambda)^{\frac{1}{\delta}}\right).$$

and the derivative of γ^* w.r.t. τ is

$$\frac{\partial\gamma}{\partial\tau} = -\frac{\phi A}{\delta}$$

The derivative of γ^* w.r.t. λ is

$$\begin{aligned}\frac{\partial\gamma^*}{\partial\lambda} &= -\frac{\omega\lambda A}{\delta} - \left(\frac{1}{\delta} + 1\right)\left(1 - (\eta\lambda)^{\frac{1}{\delta}}\right) \\ &= -\frac{\omega\lambda A}{\delta} - \left(\frac{1}{\delta} + 1\right) e_t^*\phi\eta A,\end{aligned}$$

which is always negative.

4 The debt/GDP ratio

If we compute the optimal debt/capital ratio, we get

$$d\left(\frac{B_t}{k_t}\right) = \left(\frac{g_t^*}{k_t} - (1-\phi)\,\tau\,(1-e_t^*)\,A + \frac{\omega}{2}\lambda^2 A - \frac{B_t}{k_t}\left(\frac{\left(1-\frac{\omega}{2}\lambda^2 - \tau\phi\right)A + \frac{1}{\eta} - (\rho+\lambda)}{\delta} - r\right)\right)dt$$
$$+\left(\frac{B_t}{k_t} - \frac{1-\phi}{\phi}\right)\left(\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}} - 1\right)d\Pi_t.$$

The expected value of this equation is

$$\frac{1}{dt}\mathbb{E}_t\left[d\left(\frac{B_t}{k_t}\right)\right] = \left(\frac{g_t^*}{k_t} - (1-\phi)\,\tau A + \frac{\omega}{2}\lambda^2 A + \frac{1-\phi}{\phi}\lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)\left(1-\left(\frac{1}{\eta\lambda}\right)^{1-\frac{1}{\delta}}\right)\right)dt$$
$$-\frac{B_t}{k_t}\left(\frac{\left(1-\frac{\omega}{2}\lambda^2 - \tau\phi\right)A + \frac{1}{\eta} - (\rho+\lambda)}{\delta} - r + \lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)\right)dt,$$

If the coefficient of $\frac{B_t}{k_t}$ is positive, i.e.

$$\frac{\left(1-\frac{\omega}{2}\lambda^2 - \tau\phi\right)A + \frac{1}{\eta} - (\rho+\lambda)}{\delta} - r + \lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right) > 0, \qquad (14)$$

then this expected value is a mean-reverting process and the debt/capital ratio converges towards the level

$$b^* := \frac{\frac{g_t^*}{k_t} - (1-\phi)\,\tau A + \frac{\omega}{2}\lambda^2 A + \frac{1-\phi}{\phi}\lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)\left(1-\left(\frac{1}{\eta\lambda}\right)^{1-\frac{1}{\delta}}\right)}{\frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{1}{\eta}-(\rho+\lambda)}{\delta} - r + \lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)}.$$

When there is no evasion (i.e. $1 = \lambda\eta$), the expected dynamics of debt/capital ratio is

$$\frac{1}{dt}\mathbb{E}_t\left[d\left(\frac{B_t}{k_t}\right)\right] = \left(\frac{g_t^*}{k_t} - (1-\phi)\,\tau A + \frac{\omega}{2}\lambda^2 A\right)dt$$
$$-\frac{B_t}{k_t}\left(\frac{\left(1-\frac{\omega}{2}\lambda^2 - \tau\phi\right)A - \rho}{\delta} - r\right)dt,$$

which means that this ratio is mean-reverting if the all economy grows at a rate higher than the interest rate r[1]. For the general case, although we can obtain a closed form solution for the problem, it is difficult to determine the relationship among parameters because they are highly non linear. However, for a set of parameters, it is always possible to obtain a solution.

[1]See (Levaggi and Menoncin, 2020) for a formal proof

In this case the problem for Central Government would be to find the optimal level of the tax rate and the audit parameters that maximise growth:

$$\max_{\tau,\lambda} \frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A-\rho+\frac{1}{\eta}-\lambda}{\delta}-\lambda\left(1-(\eta\lambda)^{\frac{1}{\delta}}\right)$$

subject to

$$\frac{\frac{g_t^*}{k_t}-(1-\phi)\,\tau A+\frac{\omega}{2}\lambda^2 A+\frac{1-\phi}{\phi}\lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)\left(1-\left(\frac{1}{\eta\lambda}\right)^{1-\frac{1}{\delta}}\right)}{\frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{1}{\eta}-(\rho+\lambda)}{\delta}-r+\lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)}=b^*$$

where

$$\frac{g_t^*}{k_t}=\frac{(\rho+\lambda)\,\frac{1}{\delta}+\frac{\delta-1}{\delta}\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{\delta-1}{\delta}\frac{1}{\eta}-\lambda\,(\eta\lambda)^{\frac{1}{\delta}-1}}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}}+1}$$

Since the maximisation and the constraint are linear in τ, it is possible to solve the constraint for this variable and substitute it back into the maximisation that can now be performed only for λ and as an unconstrained maximisation. However, given the highly non-linearity relationship among the parameters, a solution can be found only for numerical example as shown in A.1.

Let us now instead consider what would be the solution if Central Government would simply want to maximise

$$\max_{\tau,\lambda} \frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A-\rho+\frac{1}{\eta}-\lambda}{\delta}-\lambda\left(1-(\eta\lambda)^{\frac{1}{\delta}}\right),$$

s.t.

$$\frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{1}{\eta}-(\rho+\lambda)}{\delta}-r+\lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)>0,$$

Since the constraint and the objective function are decreasing in τ, if $A-\rho>r$ growth is maximised for $\tau=0$. In this case, also audit has no reason to exist since there is no tax evasion by definition. However, this solution implies the highest debt/GDP ratio.

4.1 The case of an agent with a log-utility function

In this section, we take the particular case of an agent whose utility is the weighted sum of two log functions as follows

$$U\left(c_t,g_t\right)=\alpha\ln c_t+(1-\alpha)\ln g_t.$$

Thus, with respect to the general model, we set $\delta=\beta=1$, $\alpha_c=\alpha$, and $\alpha_g=1-\alpha$. Under these hypotheses, the optimal evasion and consumption are

$$e_t^* = \frac{1}{\phi\eta\tau A}(1-\eta\lambda), \tag{15}$$

$$g_t^* = k_t\rho(1-\alpha), \tag{16}$$

$$c_t^* = k_t\alpha\rho. \tag{17}$$

For a log-utility consumption and public expenditure are a fixed ratio of the income flow, as one may expect. It is interesting to note that the higher the ρ, the higher consumption and hence the lower savings. Again this result is in line with the literature: if discounting is sufficiently high, consumers will increase present consumption since the return of savings is not particularly high. On the contrary, evasion depends the fiscal parameters and on the level of productivity of the economy, not on the discount rate. The expected growth rate is

$$\gamma^* := \frac{1}{dt}\mathbb{E}_t\left[\frac{dk_t}{k_t}\right] = \left(1 - \frac{\omega}{2}\lambda^2 - \tau\phi\right)A - \rho + \frac{1}{\eta} - 2\lambda + \eta\lambda^2,$$

and, finally, the expected evolution of the debt/GDP ratio

$$\frac{1}{dt}\mathbb{E}_t\left[d\left(\frac{B_t}{k_t}\right)\right] = \left(\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A-\rho-r\right)\left[\frac{\rho(1-\alpha)-(1-\phi)\tau A+\frac{\omega}{2}\lambda^2 A}{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A-\rho-r} - \frac{B_t}{k_t}\right]dt.$$

The equation above does not depend on tax evasion, i.e. the growth of the debt to GDP ratio is independent of the level of tax evasion. This effect can be explained as follows: less risk-averse individuals ($\delta = 1$) evade more, but since they incur into an higher risk (in terms of income loss following an audit) they also save more. The two effects (more debt due to less tax revenue and more GDP due to savings) perfectly compensate and the ratio is not affected by tax evasion. In other words, the numerator of the ratio increases because of tax evasion, but also the denominator increases and the ratio is not affected.

This differential equation is mean reverting if

$$\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A-\rho-r>0, \tag{18}$$

which depends on the fiscal parameters, but not on the tax evasion. The debt/GDP ratio converges towards the equilibrium value

$$b^* = \frac{\rho(1-\alpha)-(1-\phi)\tau A+\frac{\omega}{2}\lambda^2 A}{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A-\rho-r}. \tag{19}$$

The problem of the government is to maximize the expected capital growth rate under the constraint that the debt reaches a given level b^*:

$$\max_{\tau,\lambda}\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A-\rho+\frac{1}{\eta}-2\lambda+\eta\lambda^2,$$

subject to the constraint (19). This problem has an interior solution if the TFP is sufficiently high:

$$A > \frac{2\eta}{\omega}\left(1 - \phi - b^{*}\phi\right).$$

Under this condition the optimal solution is

$$\lambda^{*} = \frac{1}{\eta - \frac{1}{1-\phi-b^{*}\phi}\frac{\omega}{2}A}$$

which also implies that the fine should set to a level high enough for the intensity of the controls to be positive, i.e. $\eta > 1 - \phi\left(1 + b^{*}\frac{\omega}{2}A\right)$. Audit intensity is negatively correlated with fines. The optimal tax rate given by

$$\tau^{*} = \frac{1}{A}\frac{\rho\left(1-\alpha\right) - b^{*}\left(A - \rho - r\right)}{1 - \phi - b^{*}\phi} + \frac{1 + b^{*}}{1 - \phi - b^{*}\phi}\frac{\omega}{2}\lambda^{2}$$

$$\tau^{*} = \frac{1}{A}\frac{\rho\left(1-\alpha\right) - b^{*}\left(A - \rho - r\right)}{1 - \phi - b^{*}\phi} + \frac{1 + b^{*}}{1 - \phi - b^{*}\phi}\frac{\omega}{2}\left(\eta - \frac{1}{1 - \phi - b^{*}\phi}\frac{\omega}{2}A\right)^{-2}$$

As a special case, let us consider an economy where Government simply wants to maximise growth and where the only constraint is that the debt to GDP ratio is mean reverting. In this case, the optimal level of tax rate is zero and of course the audit and the fines are all set to zero. The debt to GDP ratio i this case is equal to:

$$b^{*} = \frac{\rho\left(1-\alpha\right)}{A - \rho - r}. \tag{20}$$

5 Conclusions

Tax evasion decisions have an important intertemporal dimension that the traditional literature has been ignoring until the recent past. In this paper we have studied the effect of debt and tax evasion on the economy in a setting where public expenditure finances consumption goods. We show that if consumers have a log-utility function, the debt to GDP ratio is not affected by tax evasion; in this case the process is mean reverting (i.e. in the long run the ratio reaches a stable equilibrium) under very soft conditions, in fact the condition is that the economic growth is higher than the interest rate. Since economic growth is itself a function of fiscal parameters the equilibrium can be reached provided that the TFP net of the discount rate is higher than the interest rate. In this context, the level of the debt to GDP ratio determines the other fiscal parameters (tax rate and audit) since it would be in theory possible to maximise growth without levying any taxes.

The study of the relationship between tax evasion, public debt and growth will have to be further extended. For the time being we have run some simulations showing that the effects may be either positive or negative. The next step is to find either an analytical solution for the effect of the debt hold by non resident investors or some simulations using real data. Another avenue we would like to explore is to find from CG point of view the optimal level of debt to be sold abroad. Finally in this model we have considered a fixed interest rate. Marakbi and Villieu (2018) uses a risk adjusted interest rate which will certainly be more realistic.

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A Proof of Proposition 2

If we call $J_t(k_t)$ the value function solving problem (7), the it must solve the following differential Hamilton-Jacobi-Bellman (HJB) equation

$$\begin{aligned}0 &= \frac{\partial J_t}{\partial t} - (\rho+\lambda) J_t + \frac{\partial J_t}{\partial k_t}\left(1-\frac{\omega}{2}\lambda^2 - \tau\phi\right) A k_t \\ &+ \max_{c_t, g_t}\left[\frac{\chi}{1-\delta}\left(\alpha_c c_t^{1-\beta} + \alpha_g g_t^{1-\beta}\right)^{\frac{1-\delta}{1-\beta}} - \frac{\partial J_t}{\partial k_t}(c_t+g_t)\right] \\ &+ \max_{e_t}\left[\frac{\partial J_t}{\partial k_t}\phi\tau e_t A k_t + \lambda J_t (k_t - \phi\eta e_t A k_t)\right],\end{aligned}$$

whose boundary condition is

$$\lim_{T\to\infty} J_T(k_T) e^{-\rho T} = 0.$$

The guess function for J_t is

$$J_t = F^{\delta}\frac{k_t^{1-\delta}}{1-\delta},$$

where F is a constant that must be computed for solving the HJB equation.

Once this form is substituted into the HJB, we get

$$
\begin{aligned}
0 = & -(\rho+\lambda) F^{\delta} \frac{k_t^{1-\delta}}{1-\delta} + F^{\delta} k_t^{1-\delta}\left(1-\frac{\omega}{2}\lambda^2 - \tau\phi\right) A \\
& + \max_{c_t, g_t}\left[\frac{\chi}{1-\delta}\left(\alpha_c c_t^{1-\beta} + \alpha_g g_t^{1-\beta}\right)^{\frac{1-\delta}{1-\beta}} - F^{\delta} k_t^{-\delta}\left(c_t + g_t\right)\right] \\
& + \max_{e_t}\left[F^{\delta} k_t^{1-\delta}\phi\tau e_t A + \lambda F^{\delta}\frac{\left(k_t - \phi\eta e_t A k_t\right)^{1-\delta}}{1-\delta}\right].
\end{aligned}
$$

The First Order Condition (FOC) on evasion gives

$$
e_t^* = \frac{1}{\phi\eta A}\left(1-\left(\frac{\eta\lambda}{\tau}\right)^{\frac{1}{\delta}}\right).
$$

The FOC on c_t is

$$
\frac{\chi}{1-\delta}\frac{1-\delta}{1-\beta}\left(\alpha_c c_t^{1-\beta} + \alpha_g g_t^{1-\beta}\right)^{\frac{1-\delta}{1-\beta}-1}\alpha_c\left(1-\beta\right) c_t^{-\beta} - F^{\delta} k_t^{-\delta} = 0,
$$

and the FOC on g_t is

$$
\frac{\chi}{1-\delta}\frac{1-\delta}{1-\beta}\left(\alpha_c c_t^{1-\beta} + \alpha_g g_t^{1-\beta}\right)^{\frac{1-\delta}{1-\beta}-1}\alpha_g\left(1-\beta\right) g_t^{-\beta} - F^{\delta} k_t^{-\delta} = 0.
$$

If we solve the system of the two optimal consumption, their relationship is

$$
c_t = g_t\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}},
$$

and, accordingly

$$
g_t^* = \chi^{\frac{1}{\delta}}\alpha_g^{\frac{1}{\delta}\frac{1-\delta}{1-\beta}}\left(\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} + 1\right)^{-\frac{1}{\delta}\left(1-\frac{1-\delta}{1-\beta}\right)}\frac{k_t}{F},
$$

$$
c_t^* = \left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}}\chi^{\frac{1}{\delta}}\alpha_g^{\frac{1}{\delta}\frac{1-\delta}{1-\beta}}\left(\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} + 1\right)^{-\frac{1}{\delta}\left(1-\frac{1-\delta}{1-\beta}\right)}\frac{k_t}{F}.
$$

The next passage is to plug e_t^*, c_t^*, and g_t^* into the HJB:

$$
\begin{aligned}
0 = & -(\rho+\lambda) F^{\delta}\frac{k_t^{1-\delta}}{1-\delta} + \chi^{\frac{1}{\delta}}\alpha_g^{\frac{1}{\delta}\frac{1-\delta}{1-\beta}}\frac{\delta}{1-\delta}\left(\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} + 1\right)^{\frac{1-\delta}{\delta}\frac{\beta}{1-\beta}} F^{\delta-1} k_t^{1-\delta} \\
& + F^{\delta} k_t^{1-\delta}\left(1-\frac{\omega}{2}\lambda^2 - \tau\phi\right) A + F^{\delta} k_t^{1-\delta}\frac{\tau}{\eta}\left(1-\left(\frac{\eta\lambda}{\tau}\right)^{\frac{1}{\delta}}\right) + \lambda F^{\delta} k_t^{1-\delta}\frac{\left(\frac{\eta\lambda}{\tau}\right)^{\frac{1-\delta}{\delta}}}{1-\delta}.
\end{aligned}
$$

The solution for F is

$$F=\frac{\chi^{\frac{1}{\delta}}\alpha_g^{\frac{1}{\delta}\frac{1-\delta}{1-\beta}}\left(\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}}+1\right)^{\frac{1-\delta}{\delta}\frac{\beta}{1-\beta}}}{\left(\rho+\lambda\right)\frac{1}{\delta}+\frac{\delta-1}{\delta}\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{\delta-1}{\delta}\frac{\tau}{\eta}-\lambda\left(\frac{\eta\lambda}{\tau}\right)^{\frac{1}{\delta}-1}}.$$

Once the value of F is substituted into the optima consumption, the result of the proposition is obtained.

***

The optimal debt

$$dB_t=\left(B_tr+g_t^*-(1-\phi)\,\tau\,(1-e_t^*)\,Ak_t+\frac{\omega}{2}\lambda^2Ak_t\right)dt-\frac{1-\phi}{\phi}\left(1-\left(\frac{\tau}{\lambda\eta}\right)^{-\frac{1}{\delta}}\right)k_td\Pi_t$$

Inverse of capital

$$\begin{aligned}dk_t^{-1}&=-k_t^{-2}k_t\frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{\tau}{\eta}-(\rho+\lambda)}{\delta}dt\\&+\left[\left(k_t-\left(1-\left(\frac{\tau}{\lambda\eta}\right)^{-\frac{1}{\delta}}\right)k_t\right)^{-1}-k_t^{-1}\right]d\Pi_t\end{aligned}$$

$$\frac{dk_t^{-1}}{k_t^{-1}}=-\frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{\tau}{\eta}-(\rho+\lambda)}{\delta}dt+\left(\left(\frac{\tau}{\lambda\eta}\right)^{\frac{1}{\delta}}-1\right)d\Pi_t$$

Debt/Capital

$$\begin{aligned}d\left(B_tk_t^{-1}\right)&=k_t^{-1}dB_t+B_tdk_t^{-1}+dB_tdk_t^{-1}\\&=\left(\frac{B_t}{k_t}r+\frac{g_t}{k_t}-(1-\phi)\,\tau\,(1-e_t^*)\,A+\frac{\omega}{2}\lambda^2A\right)dt\\&-\frac{B_t}{k_t}\frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{\tau}{\eta}-(\rho+\lambda)}{\delta}dt\\&+\frac{B_t}{k_t}\left(\left(\frac{\tau}{\lambda\eta}\right)^{\frac{1}{\delta}}-1\right)d\Pi_t-\frac{1-\phi}{\phi}\left(\left(\frac{\tau}{\lambda\eta}\right)^{\frac{1}{\delta}}-1\right)d\Pi_t\end{aligned}$$

$$\begin{aligned}\frac{1}{dt}\mathbb{E}_t\left[d\left(\frac{B_t}{k_t}\right)\right]&=\frac{g_t^*}{k_t}-(1-\phi)\,\tau\,(1-e_t^*)\,A+\frac{\omega}{2}\lambda^2A-\frac{1-\phi}{\phi}\lambda\left(\left(\frac{\tau}{\lambda\eta}\right)^{\frac{1}{\delta}}-1\right)\\&-\frac{B_t}{k_t}\left(\frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{\tau}{\eta}-(\rho+\lambda)}{\delta}-\lambda\left(\left(\frac{\tau}{\lambda\eta}\right)^{\frac{1}{\delta}}-1\right)-r\right)\end{aligned}$$

$$\frac{1}{dt}\mathbb{E}_t\left[d\left(\frac{B_t}{k_t}\right)\right] = \frac{g_t^*}{k_t} - (1-\phi)\,\tau\,(1-e_t^*)\,A + \frac{\omega}{2}\lambda^2 A - \frac{1-\phi}{\phi}\lambda\left(\left(\frac{\tau}{\lambda\eta}\right)^{\frac{1}{\delta}} - 1\right)$$

$$- \frac{B_t}{k_t}\left(\gamma^* - r + \lambda\left(\left(\frac{\tau}{\lambda\eta}\right)^{-\frac{1}{\delta}} - 1\right)\left(\left(\frac{\tau}{\lambda\eta}\right)^{\frac{1}{\delta}} - 1\right)\right)$$

A.1 Optimal τ and λ setting by Central Government

$$\max_{\tau,\lambda} \frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A - \rho + \frac{1}{\eta_1} - \lambda}{\delta} - \lambda\left(1-(\eta_1\lambda)^{\frac{1}{\delta}}\right)$$

subject to

$$\frac{\frac{g_t^*}{k_t} - (1-\phi)\,\tau A + \frac{\omega}{2}\lambda^2 A + \frac{1-\phi}{\phi}\lambda\left(1-\left(\frac{1}{\lambda\eta_1}\right)^{\frac{1}{\delta}}\right)\left(1-\left(\frac{1}{\eta_1\lambda}\right)^{1-\frac{1}{\delta}}\right)}{\frac{\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A+\frac{1}{\eta_1}-(\rho+\lambda)}{\delta} - r + \lambda\left(1-\left(\frac{1}{\lambda\eta_1}\right)^{\frac{1}{\delta}}\right)} = b^*$$

$$\frac{g_t^*}{k_t} = \frac{(\rho+\lambda)\,\frac{1}{\delta} + \frac{\delta-1}{\delta}\left(1-\frac{\omega}{2}\lambda^2-\tau\phi\right)A + \frac{\delta-1}{\delta}\frac{1}{\eta_1} - \lambda\,(\eta_1\lambda)^{\frac{1}{\delta}-1}}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} + 1}$$

***

$$\frac{\frac{(\rho+\lambda)\frac{1}{\delta}+\frac{\delta-1}{\delta}\left(1-\frac{\omega}{2}\lambda^2\right)A+\frac{\delta-1}{\delta}\frac{1}{\eta}-\lambda(\eta\lambda)^{\frac{1}{\delta}-1}}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}}+1} + \frac{\omega}{2}\lambda^2 A + \frac{1-\phi}{\phi}\lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)\left(1-\left(\frac{1}{\eta\lambda}\right)^{1-\frac{1}{\delta}}\right) - \left(\frac{\frac{\delta-1}{\delta}\phi}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}}+1} + (1-\phi)\right)}{\frac{\left(1-\frac{\omega}{2}\lambda^2\right)A+\frac{1}{\eta}-(\rho+\lambda)}{\delta} - r + \lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right) - \tau\frac{\phi A}{\delta}}$$

$$f(\lambda) := \frac{(\rho+\lambda)\,\frac{1}{\delta} + \frac{\delta-1}{\delta}\left(1-\frac{\omega}{2}\lambda^2\right)A + \frac{\delta-1}{\delta}\frac{1}{\eta} - \lambda\,(\eta\lambda)^{\frac{1}{\delta}-1}}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}} + 1} + \frac{\omega}{2}\lambda^2 A + \frac{1-\phi}{\phi}\lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)\left(1-\left(\frac{1}{\eta\lambda}\right)^{1-\frac{1}{\delta}}\right)$$

$$g(\lambda) := \frac{\left(1-\frac{\omega}{2}\lambda^2\right)A + \frac{1}{\eta} - (\rho+\lambda)}{\delta} - r + \lambda\left(1-\left(\frac{1}{\lambda\eta}\right)^{\frac{1}{\delta}}\right)$$

$$\tau = \frac{1}{A}\frac{b^* g(\lambda) - f(\lambda)}{b^*\frac{\phi}{\delta} - \frac{\frac{\delta-1}{\delta}\phi}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}}+1} - (1-\phi)}$$

$$\max_{\tau,\lambda} \frac{\left(1-\frac{\omega}{2}\lambda^2\right)A - \rho + \frac{1}{\eta_1} - \lambda}{\delta} - \lambda\left(1-(\eta_1\lambda)^{\frac{1}{\delta}}\right) - \frac{\phi A}{\delta}\tau$$

$$\max_{\lambda} \frac{\left(1-\frac{\omega}{2}\lambda^2\right)A - \rho + \frac{1}{\eta} - \lambda}{\delta} - \lambda\left(1-(\eta_1\lambda)^{\frac{1}{\delta}}\right) - \frac{b^* g(\lambda) - f(\lambda)}{b^* - \frac{\delta-1}{\left(\frac{\alpha_c}{\alpha_g}\right)^{\frac{1}{\beta}}+1} - \delta\frac{1-\phi}{\phi}}$$

FOC

$$\frac{\left(1-\frac{\omega}{2}\lambda^{2}\right)A-\rho+\frac{1}{\eta}-\lambda}{\delta}-\lambda\left(1-(\eta\lambda)^{\frac{1}{\delta}}\right)-\frac{b^{*}g\left(\lambda\right)-f\left(\lambda\right)}{b^{*}-\frac{\delta-1}{\left(\frac{\alpha_{c}}{\alpha_{g}}\right)^{\frac{1}{\beta}}+1}-\delta\frac{1-\phi}{\phi}}$$