The Supply and Demand of Tax Avoidance

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Abstract

We recognise the existence of a competitive 'tax advice' industry supplying tax avoidance schemes which help taxpayers reduce their tax liability. We model both the demand and the supply sides of this market. It is assumed that there is a continuum of risk-averse taxpayers buying such schemes from a firm at per unit market price, subject to a minimum investment induced by the existence of the customer-specific set-up cost. The tax authority may mount a legal challenge to these schemes and, if successful, it can only reclaim the tax owed but cannot levy a fine. The Firm faces a legal cost if schemes are challenged by the tax authority and has other fixed costs. Under these assumptions, we find that (1) the individual's demand for avoidance is a function of wealth: fettered may choose to avoid an amount $\frac{\tau}{p}$ or all his wealth and unfettered taxpayers avoid the optimal proportion of their income $\mathcal{A}$ at the per unit market price; (2) the unit price of the tax avoidance scheme is affected by the tax rate, the probability of successful legal challenge, and the proportion of avoided income to total income; (3) tax avoidance of unfettered taxpayers is price elastic; (4) so in equilibrium, both fettered and unfettered taxpayers avoid all their wealth, but fettered taxpayers bear a higher per unit price to avoid tax than unfettered taxpayers in general; (5) the extensive margin of fettered taxpayers is greater than the intensive margin of unfettered taxpayers; (6) by simulation, we find the Laffer curve indicating the tax authority should make appropriate tax rate if the goal is maximising tax revenue.

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1 Introduction

Tax avoidance is a significant economic problem, which causes great losses to tax revenue. From 2017 to 2018, the value of the tax gap – the difference between the theoretical tax liability and the actual amount of tax paid – was estimated by the UK tax authority to be £35 billion, which was 5.6% of tax liabilities and £1.8 billion was caused by tax avoidance (H.M. Revenue and Customs, 2019). Individuals take a variety of actions to reduce their tax liabilities and the UK tax authority defines three different types of behaviour (H.M. Revenue and Customs, 2012): (i) tax avoidance is exploiting the tax rules to gain a tax advantage that lawmakers never intended; (ii) tax evasion is an illegal activity, where registered individuals or businesses deliberately omit, conceal or misrepresent information in order to reduce their tax liabilities; (iii) tax planning involves using tax reliefs for the purpose intended by lawmakers.

We recognise the existence of a competitive "tax advice" industry providing schemes which help taxpayers reduce their tax liability. In this paper, we model both sides of the tax avoidance market. For the demand side, we develop the portfolio model of evasion and avoidance (Gamannossi degl'Innocenti and Rablen, 2017) to analyse whether taxpayers avoid their tax liabilities when they are allowed to buy tax avoidance schemes in the market and get the aggregate tax avoidance function. For the supply side, we assume that a sigle firm makes profits by selling the tax avoidance scheme to taxpayers who can afford the minimum avoidance fee. Combining the demand function and supply function, we get the equilibrium solutions and economic insights in seeking to understand tax avoidance behaviour.

The first economic studies relating to tax non-compliance mainly discuss tax evasion. Allingham and Sandmo (1972) present the canonical portfolio model of tax evasion decision in which individual taxpayers choose whether to evade and, if so, how much to evade under uncertainty. There is a trade-off between a gain if the evasion is undetected and a loss if the evasion is detected and penalised taking account of the tax rate, the probability of detection and the penalty rate imposed on evaded income. Ambiguous results are derived for declared income and tax rate because of the risk aversion types and the income and substitution effects; but unambiguous results are also derived that an increase in the penalty rate and the probability of detection will lead to a larger amount of declared income. In addition, if the fine is imposed on evaded tax, then the evaded income decreases as the tax rate increases because there is only an income effect, which is called the 'Yitzhaki Puzzle'. Under the assumption of penalty being a function of the proportion of understated income to actual income and the probability of detection being independent of the level of income, Srinivasan (1973) shows that, compared with proportional tax structure, progressive tax function results in more losses in tax revenue due to tax evasion. Subsequent theoretical work has extended the basic analysis to consider alternative penalty and tax functions and endogenous income (Yitzhaki, 1974; Pencavel,1979; Cowell, 1985). Ambiguities in the theoretical results have led to numerical and econometric analysis of individual tax evasion behaviour[1]. Study of individual-choice problem has also generated work on optimal government choice of taxes, penalties, and probabilities of detection in a world with tax evasion [2].

Avoidance models follow evasion models. Alm (1988) recognises that there is another legal channel for tax reduction —– tax avoidance. He analyses individual behaviour when avoidance and evasion are simultaneously available and government behaviour when individuals have these options. In his model, the individual is allowed to report, avoid and evade income given the fixed endowment of income; tax avoidance activity is riskless but has a participation cost while evasion is risky. This is because the individual might be audited by the tax authorities with a fixed probability and if detected, the individual will be fined an amount which is subject to the amount of evasion income. The marginal tax rate, the participation cost function of avoidance and the penalty function of evasion are positive and increasing. The avoidance choice alters many of the conclusions of the simpler evasion literature. First, higher probability of detection and an increase in the marginal penalty cost and the marginal tax rate could decrease the amount of evasion but do not mean that the tax base will increase as there are now two channels to reduce tax liabilities and evasion can flow to avoidance. Second, the cost function of avoidance plays an important role, for example, if the marginal cost is decreasing, then an increase in the probability of detection unambiguously reduces taxable income even though evasion declines. Third, social welfare maximisation leads government to set its instruments at lower levels than when it is only interested in net revenues maximisation, and the government gains tax revenues from tax complexity because the size of the tax base increases with greater complexity. Alm and McCallin (1990) apply the portfolio theory (return-mean and risk-variance) to the avoidance-evasion decision and consider both avoidance and evasion as risky activities. Given this assumption, a different conclusion is drawn that an increase in the fine rate increases the taxable income but similar

[1] Friedland et al. (1978) for a simulation study of evasion; Econometric analysis of evasion behaviour is performed by Clotfelter (1984), Slemrod (1985), and Witte and Woodbury (1985).

[2] See Singh (l973), Christiansen (1980), Sandmo (l981), and Polinsky and Shavell (1984). The work by Graetz et al. (1986) and Reinganum and Wilde (1985) analyzes optimal government policy in a world in which the individuals and the collection agency interact with one another in a game theory context.

conclusions are also drawn such as the government gains from tax complexity. Econometric analysis of tax compliance behaviour has been performed as well. Alm et al. (1990) estimate individuals' tax compliance behaviour including evasion and avoidance by using the Tobit maximum likelihood estimation based on the individual-level data in Jamaica in 1983, which take account the payroll tax contributions and benefits. The results indicate that evasion and avoidance are substitutes and both are effective vehicles for reduction in tax liability. So better enforcement will not necessarily increase the tax base and the tax base rises with higher benefits for payroll tax contributions.

Most of the previous theoretical work allow taxpayers to make decisions under uncertainty while Cowell (1990) analyses the cost of sheltering and evasion using a certainty-equivalent model which specifies the cost-of-concealment function a priori. He concludes that the rich who are risk averse will choose the sheltering (riskless but costly) option and the poor could only choose evasion because they cannot afford the fee of tax concealment schemes; under these circumstances, it is the poor who end up paying the penalty for getting caught in tax evasion and it is the poor who end up paying the taxes too. Slemrod (2001) also uses the certainty-equivalent model and take labour supply into account in the avoidance model. In this model, the response to taxation can be divided into two groups: real substitution response, in which the tax-induced change in relative prices causes individuals to seek a different consumption bundle; (ii) and avoidance response, in which taxpayers take a variety of tax avoidance activities to directly reduce tax liability without consuming a different basket of good. By allowing individuals to make the labour-leisure choice and change their avoidance effort in response to tax reforms (changes), it draws the conclusion that the opportunities for tax avoidance mitigate the real substitution response to taxation. Neck et al. (2012) discuss the effects of (legal) tax avoidance and (illegal) tax evasion on the shadow economy. They build a theoretical microeconomic model in which households can participate in the official and in the shadow economy. Using comparative statics, it shows that the more complex the tax system is, the more possibilities of legal tax avoidance exist, and hence a smaller labour supply in the shadow economy. It also shows that a reduction in the maximum admissible number of working hours in the official economy increases the labour supply in the shadow economy.

Gamannossi degl'Innocenti and Rablen (2017) notice that tax avoidance schemes are marketed and many taxpayers buy these schemes at given price to reduce their tax liability. In the theoretical model, a narrow bracketing approach is used, which assumes taxpayers make the avoidance choice before evasion. The taxpayer's income declaration is audited with probability and if audited, both avoidance and evasion are observed and the taxpayer has to pay a fine on the evaded tax. Then the tax authority will mount a legal challenge to the avoidance scheme, which is successful with a certain probability. If the legal challenge is successful, the taxpayer is only asked to repay the tax owed (not a fine). The model gets the interior solutions for optimal avoidance and evasion and finds an analogy of 'Yitzhaki puzzle' for avoidance—an increase in the tax rate decreases the level of avoided income and the avoided tax. The results also show that evasion is an increasing function of the audit probability when the latter is low enough, yet tax avoidance is always decreasing in the probability of audit. And when holding the expected return to evasion constant, it is not always the case that the total loss of reported income due to avoidance and evasion can be stemmed by increasing the fine rate and decreasing the audit probability.

The above literature models the decision-making of tax avoidance and evasion, however, they only analyse the demand of reduction in tax liabilities. Slemrod (2004) considers the ignorance of the supply side of tax non-compliance to be a significant shortcoming of traditional economic models, especially in relation to corporate tax behaviour, and points out that the market for tax abusive schemes has grown substantially in recent years.[3] In related research, Damjanovic and Ulph (2010) model both demand and supply sides of the tax avoidance market under risk neutral assumption and get the equilibrium price and hence, the level of non-compliance. The primary focus of his contribution is that the flatter the tax schedule, the lower is the equilibrium price of tax minimisation schemes and hence, the greater is the level of non-compliance. The results indicate that there will be greater tax compliance in economies with a higher level of inequality in pre-tax income. And given the tax code and pre-tax income distribution, the government can design the monitoring and penalty functions to influence tax evasion and hence, the proportion of non-compliant taxpayers. There are, however, important differences with our model. Firstly, His model assumes that the taxpayer will be audited with some probability and if audited, the avoidance scheme will be deemed to constitute tax evasion in which case the taxpayer will have to repay the tax plus a penalty. However, we treat avoidance scheme as legal and so if tax authority mounts a legal challenge and succeed, the taxpayer who is risk-averse only needs to repay the tax.

[3] The large importance of the disclosure of the tax avoidance schemes has been recognised by HMRC and postulated in Part 7 of the Finance Act 2004. Similarly, the Internal Revenue Service proclaimed that one of its priorities in 2009 is to combat abusive tax avoidance schemes and the individuals who promote them (IRS, 2009).

Secondly, in his assumption, the price of the tax avoidance scheme is determined by marginal costs, competitiveness of the industry and the nature of the demand schedule, but in our model, price is determined by tax rate, the successful probability of legal challenge and the proportion of avoided income to true wealth. Recent work by Alstadsæter et al. (2019) combines micro-data leaked from financial institutions in tax havens with random audits and population-wide administrative income and wealth records in rich countries (Norway, Sweden and Denmark). It focuses on inequality problems raised by tax evasion and estimates the size and distribution of total tax evasion. The results show that tax evasion has important implications for the measurement of inequality, and compared with tax avoidance, fighting tax evasion can be a more effective way to collect more tax revenue from the very wealthy. A theoretical model is also built to describe the supply side of tax advice market, but the cost of offering such a scheme is the penalty to firms if caught breaking the law, which is not applicable to tax avoidance schemes.

Although a couple of paper models both sides of the tax avoidance market, they do not capture the characteristics properly. An important feature of our model is that it addresses explicitly the high customer-specific set-up cost when the firm offers the tax avoidance scheme to each taxpayer. Accordingly, the marginal cost of adding one more person into the scheme is significant whereas passing one more pound through the scheme that has already been set up costs almost nothing, which is different from Damjanovic and Ulph's (2010) supply model. In addition, there is a minimum wealth threshold for taxpayers induced by the customer-specific set-up cost. The firm also has legal cost if the tax authority mounts a legal challenge to the avoidance scheme. Another important feature of our model is that the price of the tax avoidance scheme is per unit price with minimum fee instead of per scheme price. So taxpayers will pay more fee to promoters if they avoid more income.

Our model is simple enough to admit an analytic solution, but it is also sufficiently rich that it discusses several implications of interest to academics and practitioners in tax authorities. First, the individual's demand for avoidance is a function of wealth, and the unit price of the tax avoidance scheme is affected by the tax rate, the probability of successful legal challenge, and the proportion of avoided income to total income. In reality, the successful probability of legal challenge is pretty high if tax authority mounts a challenge to the scheme while the probability of mounting a legal challenge is pretty low, therefore, increasing the probability of legal challenge might be an effective way to reduce avoidance. Second, tax avoidance of unfettered taxpayers is price elastic, so firms impose no upper limits on the amount that can be avoided; and in equilibrium, both fettered and unfettered taxpayers avoid all their wealth, but fettered taxpayers bear a higher per unit price to avoid tax than unfettered taxpayers in general. Third, the extensive margin of fettered taxpayers for avoidance is greater than the intensive margin of unfettered taxpayers. The last, by simulation, we find the Laffer curve indicating when the tax rate is lower than the revenue-maximising tax rate, an increase in tax rate increases both aggregate avoidance and total tax revenue; when the tax rate is higher than the revenue-maximising tax rate, increasing tax rate will not only reduce tax revenue but also increase aggregate avoidance.

The paper proceeds as follows: section 2 gives assumptions and develops a formal model of tax avoidance from the demand side and the supply side for the "tax advice" industry. Section 3 analyses the model and comparative statics of taxpayers' optimal avoidance, and get the equilibrium. Section 4 shows the simulation results, and section 5 concludes. Proofs omitted from the text are collected in the Appendix, and figures are at the very rear.

2 Model

In the demand side, there is a continuum of risk-averse taxpayers and their wealth probability density function and cumulative distribution function are $g(w)$ and $G(w)$. Each taxpayer i has an exogenously income (wealth) w_i and faces a tax on income given by tw_i, where $t \in (0, 1)$. Taxpayers behave as if they maximize expected utility, where utility is denoted by $U(z) = \log(z)$.[4] Taxpayers' true income is not observed by the tax authority and they can choose whether declare their true income but they must declare an amount $x_i \in [0, w_i]$. Taxpayers can choose to avoid paying tax on an amount of income $A_i \in [0, w_i]$, so $x_i = w_i - A_i$. Avoidance technology is, though, costly, because devising tax avoidance schemes that reduce tax liability without ostensibly violating tax law need to take full advantage of various provisions of the income tax code, coupled with a degree of ingenuity, that few taxpayers possess.[5] To satisfying this demand, a number of firms which are called "promoters" supplying and marketing avoidance schemes appear and gradually form the

[4] Thus, the risk-averse taxpayers have a constant (unit) coefficient of relative relative risk aversion. We adopt the logarithmic form for reasons of annalytic tractability, though we note that the assumption of constant relative risk aversion commands considerable empirical support (see, e.g., Chiappori and Paiella 2011).

[5] People not only have difficulties in understanding tax law and codes, but also show poor knowledge of tax rates (Blaufus *et al.*, 2015; Gideon, 2017) and basic concepts of taxation.

competitive 'tax advice' market. A common feature of this market is the "no saving, no fee" arrangement under which the avoidance fee received by a promoter is linked to the amount by which their scheme stands to reduce the user's tax liability. From a detailed investigation in the UK that, for the majority of mass-marketed schemes, the fee is related to the reduction in the annual theoretical tax liability of the user, not the *expost realisation* of the tax saved (Committee of Public Accounts, 2013). Thus, the monetary risks associated with the possible subsequent legal challenge and the termination of a tax avoidance scheme are borne by the user.

In the supply side, there are a number of firms (e.g. 50-100 active promoters in the market) and each promoter provides only one type of scheme. The five largest type of mass marketed tax avoidance schemes are Partnership Loss schemes, Employee Benefit Trust schemes, Interest Relief schemes, Employment intermediary schemes and Stamp Duty Land Tax schemes. Taking Employee Benefit Trust schemes for example to explain how tax avoidance achieved, tax advice firms set up trusts offshore and claim that the trust could help employees avoid the need to pay income tax and National Insurance contributions. They sell these schemes to employers and self-employed individuals (they are both employers and employees), charge some fees and then make loans to employees, which are not taxable. In practice, the loans are never repaid and are used as a way of rewarding employees. The nature of the Employee Benefit Trust is an disguised remuneration tax avoidance scheme. Some taxpayers buy more than one tax avoidance scheme as it is risky investment and they prefer to spread risks that found by the tax authority. However, most taxpayers avoid with only one firm because, firstly, firms have a minimum fee requirement that most taxpayers can not spread their avoided income over two or more firms. Secondly, purchasing different types of schemes will increase avoidance cost of employers (devising a new scheme is costly but adding one more people into the existing scheme cost almost nothing) and the difficulty of execution (i.e. paying employees compensation through two bank accounts is challenging and require a lot of effort for both employers and promoters). Thirdly, advisers claim Employee Benefit Trust schemes are legal when they sell the schemes to employers and self-employed individuals, and if they suggest customers for diverse schemes, promoters will lose part of fees because some taxpayers may spread avoided income with other firms. So promoters always advice customers to purchase tax avoidance schemes with only one firm. Although there are a number of tax advice firms available to taxpayers, promoters behave like monopolists. This is because the tax avoidance scheme is super complicated, and taxpayers could not understand and tell the difference. Therefore, they just choose a firm randomly, which means the selected firms act as a monopolist.

To keep things simple, we assume that, in the market, there is a single firm making and selling a type of tax avoidance schemes to taxpayers at per unit market price $p < t$ (otherwise taxpayers can not benefit from the scheme and so will not buy them anymore) with minimum avoidance fee $\underline{f}$. Accordingly, there are three types of taxpayers: the first type is those who are excluded form the market as they can not afford the minimum avoidance fee and could only choose avoid nothing; the second type is those who want to avoid income tax but are constrained by the minimum avoidance fee; the third type is those who are not constrained by the minimum avoidance fee so they could buy the scheme freely at per unit market price p, in this case, the fee received by the firm is linked to the amount that can be avoided. Therefore, their avoidance fee is 0, the minimum fee and per unit market price multiply by the amount of avoided income respectively, summarised by $f_i \in \{0, \underline{f}, pA_i\}$. We call the second and third types the fettered and unfettered taxpayers. As I mentioned in the example of Employee Benefit Trust schemes, before the tax advice firm carries out a scheme, it needs to set up a trust offshore for the taxpayer. Accordingly, the minimum avoidance fee $\underline{f}$ is arising endogenous owing to the existence of customer-specific set-up cost τ. Therefore, the marginal cost of adding one more person is significant and given by τ, but once the trust is set up, passing one more pound through the scheme that has already been set up costs almost nothing, so the minimum avoidance fee is equal to the customer-specific set-up cost, $\underline{f} = \tau$. Before setting up the trust, to provide an effective tax reduction scheme to taxpayers, the supplier must conduct complex research into local and international tax law, devise a scheme and then "test" it by seeking a legal opinion as to whether it works in law (Damjanovic and Ulph, 2010). We call the cost induced in this process as fixed cost υ.

Except for the customer-specific set-up cost and the fixed cost, the firm also has legal cost. For example, the UK government (H.M. Revenue and Customs, 2012) introduced a disclosure regime, the Disclosure of Tax Avoidance Schemes (DOTAS) regime in 2004.[6] DOTAS requires promoters who design and sell certain types of avoidance schemes to disclose information about the schemes to HMRC. Taxpayers who use such a scheme are also required to report the scheme reference number on their tax return. The DOTAS rules have been expanded over time and if promoters and users do not report to HMRC, they will face huge penalty up to £1 million per scheme (Finance Act, 2010). DOTAS is intended to capture information about marketed avoidance schemes, but is not restricted to marketed schemes.

[6]DOTAS excludes VAT. There is a separate disclosure regime for VAT, which was introduced in 2004.

The tax authority could see the schemes through DOTAS and does not need to conduct an audit. It will mount a legal challenge to some of these schemes, accordingly promoters have relevant legal cost in order to deal with enquiries from HMRC. Compared to the number of users of avoidance schemes, only a small number of cases enter litigation. That could be explained by 'lead case' by National Audit Office — one case, or a small group of cases, will be litigated as a lead case, with the judgment intended to resolve a group of similar cases. Where it considers it feasible, HMRC may ask the Tax Tribunal to apply a ruling (Rule 18) to bind a group of follower cases to accept the judgment of a lead case, subject to any subsequent appeal to distinguish the related cases. Although only a small number of cases enter litigation, HMRC has a high success rate when it litigates avoidance cases. If users are deemed as avoidance, they need to pay the due tax. Given the above information, we assume the firm faces a probability of a legal challenge ρ_L that tax authority may mount to the avoidance scheme, and if challenged, it has a relevant legal cost c_L to deal with enquiries from the tax authority. The legal challenge is successful with probability ρ_s and so the probability that the scheme is challenged successfully is $\rho = \rho_L \rho_s$. If the legal challenge is successful, the tax authority obtains the right to reclaim the tax owned from taxpayers and shut down the scheme but cannot levy a fine on taxpayers and the firm. In this case, instead of paying tx_i in tax, the taxpayer must pay tw_i and the tax advice firm (industry) will go bankruptcy because there is only one firm making and selling a type of tax avoidance scheme in our model.

Given the above assumptions, the expected utility of taxpayer i is

$$EU(A_i) = \rho U(w_i^s) + (1-\rho) U(w_i^u) \tag{1}$$

where w_i^s is the i^{th} taxpayer's wealth when the tax authority mounts a legal challenge and succeeds and w_i^u is the i^{th} taxpayer's wealth when avoidance succeeds:

$$\begin{aligned} w_i^s &= w^s(w_i, f_i) = (1-t) w_i - f_i; &(2)\\ w_i^u &= w^u(w_i, A_i, f_i) = (1-t) w_i + tA_i - f_i. &(3) \end{aligned}$$

f_i is the piecewise function of avoidance fee for different taxpayers:

$$f_i = \begin{cases} pA_i & \text{if } f_i > \underline{f} \\ \tau & \text{if } f_i = \underline{f} \\ 0 & \text{otherwise} \end{cases} . \tag{4}$$

And the firm's expected profit function is given by

$$E(\pi) = \int f_i g(w)\ \mathrm{d}w - \tau \int \mathbf{1}_{A_i(w)>0} g(w)\ \mathrm{d}w - c_L \rho_L - \upsilon,$$

where $\mathbf{1}_{A_i(w)>0}$ is a dummy variable and

$$\mathbf{1}_{A_i(w)>0} = \begin{cases} 1 & \text{if } A_i(w) > 0 \\ 0 & \text{otherwise} \end{cases}$$

which indicates avoidance when it takes 1 and no avoidance when it takes 0.

3 Analysis

For an unfettered taxpayer, differentiating (1) with respect to f_i we have that

$$\frac{\partial EU(A_i)}{\partial f_i} = (1-\rho)\frac{t-p}{p w_i^u} - \rho \frac{1}{w_i^s}. \tag{5}$$

Solving for the point $\partial EU(A_i)/\partial f_i = 0$ gives the optimal avoidance fee f_i^*:

$$f_i^* = \frac{(1-t)(t-p-\rho t)}{t-p} w_i. \tag{6}$$

As $f_i = pA_i$, we get the optimal avoided income A_i^*:

$$A_i^* = \frac{(1-t)(t-p-\rho t)}{p(t-p)} w_i. \tag{7}$$

We can see from equation 7, the optimal avoided income A_i^* is proportional to the wealth of the unfettered taxpayer. So We define

$$\mathcal{A} \equiv \frac{A_i^*}{w_i} = \frac{(1-t)(t-p-\rho t)}{p(t-p)} \tag{8}$$

is the optimal proportion of avoidance which is constant; where

$$p + \rho t < t; \qquad t(1-t)(1-\rho) \leqslant p(1-p). \tag{9}$$

The left-side inequality guarantees that $\mathcal{A} > 0$, and the right-side inequality ensures that $\mathcal{A} \leqslant 1$.

Rearranging equation 8 we obtain a quadratic in p

$$g(p) = \mathcal{A}p^2 - (\mathcal{A}t + 1 - t)p + t(1-t)(1-\rho) = 0 \tag{10}$$

Lemma 1 *The inverse demand function is given by*

$$p = t - \frac{\mathcal{A}t - (1-t) + \sqrt{[\mathcal{A}t - (1-t)]^2 + 4\mathcal{A}\rho(1-t)t}}{2\mathcal{A}} \tag{11}$$

Lemma 1 is from solving equation 10 for p and the proof is in the Appendix. We see that the price of avoidance schemes is affected by the tax rate, the probability of successful legal challenge, and the proportion of avoided income to total income.

Lemma 2 *At the optimal avoidance for unfettered taxpayers, it holds for $\mathcal{A}$ that*

$$\frac{\partial \mathcal{A}}{\partial p} = \frac{2\mathcal{A}p - \mathcal{A}t - (1-t)}{p(t-p)} < 0; \qquad \frac{\partial \mathcal{A}}{\partial \rho} = -\frac{t\,(1-t)}{p\,(t-p)} < 0 \tag{12}$$

$$\frac{\partial \mathcal{A}}{\partial t} = \frac{(1-\mathcal{A})\,p + (1-2t)\,(1-\rho)}{p(t-p)} > 0 \text{ if } t < 0.5 \tag{13}$$

Lemma 2 is derived via implicit differentiation of the equation (10), so we omit the proof. It shows that an increase in price of avoidance scheme and the probability of successful legal challenge by the tax authority to the avoidance scheme will make unfettered taxpayers decrease the optimal proportion of avoidance. In reality, the tax rate is less than 50%. For a given price of avoidance scheme p and the probability of successful legal challenge to avoidance scheme ρ, an increase in tax rate will make unfettered taxpayers avoid more as they can benefit more.

For the unfettered taxpayer, his avoided income A_i is greater than $\frac{\tau}{p}$ and the best choice is avoiding the amount A_i^*, which implies there is a critical value of wealth $\tilde{w}_1$ (we call it the upper bound of wealth for fettered taxpayers) and when $w_i > \tilde{w}_1$, $A_i = A_i^* > \frac{\tau}{p}$ holds. Follows equation (7) and (8), we get

$$w_i > \tilde{w}_1 \equiv \tilde{w}_1(\mathcal{A}) = \frac{t-p}{(1-t)(t-p-\rho t)}\tau = \frac{\tau}{p\mathcal{A}}. \tag{14}$$

Therefore, if an individual's income is greater than $\tilde{w}_1$, the optimal amount of avoidance is $A_i^* = \mathcal{A}w_i$. $\tilde{w}_1$ is increasing in the customer-specific set-up cost and decreasing in the per unit market price and the proportion of avoided income to true income. Rearranging equation 14 we get

$$z = \tilde{w}_1 - \frac{\tau}{p\mathcal{A}} = 0 \tag{15}$$

Lemma 3 *At the optimal avoidance for unfettered taxpayers, it holds for $\tilde{w}_1$ that*

$$\frac{\partial \tilde{w}_1}{\partial p} = -\frac{\tilde{w}_1}{p} < 0; \qquad \frac{\partial \tilde{w}_1}{\partial \mathcal{A}} = -\frac{\tilde{w}_1}{\mathcal{A}} < 0; \qquad \frac{\partial \tilde{w}_1}{\partial t} = -\frac{\tilde{w}_1 \mathcal{A}_t}{\mathcal{A}} < 0 \text{ if } t < 0.5 \tag{16}$$

$$\frac{\partial \tilde{w}_1}{\partial \rho} = -\frac{\tilde{w}_1 \mathcal{A}_\rho}{\mathcal{A}} > 0; \qquad \frac{\partial \tilde{w}_1}{\partial \tau} = \frac{\tilde{w}_1}{\tau} > 0 \tag{17}$$

Lemma 3 is derived via implicit differentiation of the equation 15, so we omit the proof. It shows that the upper bound of wealth for fettered taxpayers is decreasing in the per unit market price, the optimal proportion of avoidance and the tax rate but increasing in the probability of successful legal challenge and the customer-specific set-up cost.

For a fettered taxpayer, he is constrained by the minimum avoidance fee and so could only choose to avoid an amount exactly equal to $\frac{\tau}{p}$ or all his wealth, $A_i = \left\{\frac{\tau}{p}, w_i\right\}$[7], so that $f_i = \tau$. Other taxpayers are excluded form the market as they can not afford the minimum avoidance fee, $f_i = 0$, and could only choose to avoid nothing, $A_i = 0$. In this case, there is a cut-off point of wealth for avoidance $w_i = \tilde{w}_0$ such that a taxpayer's expected utility is indifferent between choosing either. This can be expressed as follows:

$$U\left[w^s\left(\tilde{w}_0, 0\right)\right] = \rho U\left[w^s\left(\tilde{w}_0, \tau\right)\right] + \left(1-\rho\right) U\left[w^u\left(\tilde{w}_0, A_i, \tau\right)\right]. \tag{18}$$

Lemma 4 *At the optimal avoidance for fettered taxpayers, it holds for $\tilde{w}_0$ that*

$$\frac{\partial \tilde{w}_0}{\partial p} \geq 0; \qquad \frac{\partial \tilde{w}_0}{\partial \rho} > 0; \qquad \frac{\partial \tilde{w}_0}{\partial \tau} > 0 \tag{19}$$

Lemma 3 is obtained via implicit differentiation from equation (18) and the proof is in Appendix. It clarifies that the cut-off point of wealth for avoidance is increasing in the per unit market price, the probability of successful legal challenge and the customer-specific set-up cost.

[7] At this time, he may be only constrained by the minimum fee, in this case, the fettered taxpayer will choose to avoid a fixed amount $A_i = \frac{\tau}{p}$ of his income at the market unit price p if avoidance is better off than no avoidance even though he avoids greater proportion of his income compared with unfettered taxpayers. He may be constrained by his wealth as well, in this case, the minimum avoidance fee of the fettered taxpayer converts into avoidance up until avoidance reaches the total wealth, in other words, the fettered taxpayer avoids all his wealth $A_i = w_i$ bearing a higher price than the per unit market price as long as avoidance is better off than no avoidance. These two cases generate two cut-off points of wealth of fettered taxpayers $\tilde{w}_0 = \left\{\tilde{w}_0^{'}, \tilde{w}_0^{''}\right\}$, we discuss both cases $A_i = \frac{\tau}{p}$ and $A_i = w_i$ in Appendix.

Proposition 1 *The individual's demand for avoidance is a function of wealth:*

$$A_i(w) = \begin{cases} \mathcal{A}w_i & \text{if } w_i > \tilde{w}_1 \\ \frac{\tau}{p} & \text{if } w_i \in \left[\frac{\tau}{p}, \tilde{w}_1\right] \\ w_i & \text{if } w_i \in \left[\tilde{w}_0, \frac{\tau}{p}\right] \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

Combining equation (14) and (18), we get Proposition 1. As fettered taxpayers may choose to avoid an amount exactly equal to $\frac{\tau}{p}$ or all his wealth, Proposition 1 has two specific piecewise functions of avoided income on wealth, we use Figure 1 to explain the case when fettered taxpayers choose to avoid all their wealth. It shows that (i) points on the horizontal axis indicate taxpayers whose income is lower than the cut-off point of wealth –$\tilde{w}_0$– are excluded from the tax avoidance market by the minimum investment requirement; (ii) points on the solid gray line indicate wealthy unfettered taxpayers, their income is higher than $\tilde{w}_1$, they avoid the optimal proportion of their income $\mathcal{A}$ at the market unit price; (iii) points on the solid black 45-degree line indicate the wealth-constrained taxpayers, their income is between $\tilde{w}_0$ and $\frac{\tau}{p}$, and this line is steeper than the solid gray line which means they have to avoid all income at a higher per unit market price so that the firm provide tax avoidance schemes at the minimum fee; (iv) points on the solid black horizontal line indicate the minimum-fee-constrained taxpayers, their income is between $\frac{\tau}{p}$ and $\tilde{w}_1$, they can avoid tax at the same per unit market price as wealthy unfettered taxpayers but they have to avoid a larger proportion than $\mathcal{A}$ which is equal to the fixed amount $\frac{\tau}{p}$. The existence of customer-specific set-up cost τ changes taxpayers' avoidance behaviour: it makes low-income taxpayers decrease their avoidance to zero (the light gray area) and fettered taxpayers increase their avoidance (the dark gray area).

Therefore, the aggregate avoidance A is given by

$$A = \int A_i(w)\, g(w)\, \mathrm{d}w. \tag{21}$$

Differentiating (21) with respect to p and ρ gives

$$\frac{\partial A}{\partial p} < 0; \qquad \frac{\partial A}{\partial \rho} < 0. \tag{22}$$

This implies that aggregate avoidance is decreasing in unit price of tax avoidance schemes and the probability that the scheme is challenged successfully.

We know that net revenue from fettered taxpayers is zero so we can write down the expected profit of the firm as follows:

$$
\begin{aligned}
E(\pi_j) &= \int_{\tilde{w}_1}^{\infty} f(w)\, g(w)\ \mathrm{d}w - \tau \int_{\tilde{w}_1}^{\infty} g(w)\ \mathrm{d}w - c_L \rho_L - \upsilon & (23) \\
&= p \int_{\tilde{w}_1}^{\infty} \mathcal{A} w g(w)\ \mathrm{d}w - \tau \left[1 - G(\tilde{w}_1)\right] - c_L \rho_L - \upsilon & (24) \\
&= p A_u - \tau \left[1 - G(\tilde{w}_1)\right] - c_L \rho_L - \upsilon & (25)
\end{aligned}
$$

where A_u is total avoidance of unfettered taxpayers and $A_u = \int_{\tilde{w}_1}^{\infty} \mathcal{A} w g(w)\ \mathrm{d}w = \mathcal{A} \mu_{w>\tilde{w}_1} \left[1 - G(\tilde{w}_1)\right]$.

Proposition 2 *The demand of tax avoidance for unfettered taxpayers is price elastic,* $\varepsilon_{A_u,p} > 1$.

Proof of Proposition 2 is in Appendix. Differentiating pointwise, the first-order condition of firm's expected profit function is given by

$$
\frac{\partial E(\pi_j)}{\partial A_u} = \frac{\partial p}{\partial Au} A_u + p = p\left(1 - \varepsilon_{p,A_u}\right) \tag{26}
$$

From proposition 2 we know $\varepsilon_{A_u,p} > 1$, so $\varepsilon_{p,A_u} \in (0,1)$ and $\frac{\partial E(\pi_j)}{\partial A_u} > 0$. This indicate that the firm prefers to decrease the price slightly to get higher demand in return as the demand of tax avoidance for unfettered taxpayers is price elastic, and for a given price, the firm would not impose upper limit on the amount that can be avoided. Therefore, we get the following proposition.

Proposition 3 *In equilibrium, both fettered and unfettered taxpayers avoid all their wealth.*

$$
A_i = \begin{cases} w_i & \text{if } w_i \geqslant \tilde{w}_0 \\ 0 & \text{otherwise} \end{cases}
$$

So $\mathcal{A} = 1$ *and* $\tilde{w}_1 = \frac{\tau}{p}$ *holds, and from (8), the optimal price of tax avoidance schemes is given by*

$$
p^* = t - \frac{1}{2}\left([2t-1] + \sqrt{[2t-1]^2 + 4\rho t\,[1-t]} \right). \tag{27}
$$

Considering equation (12), (16), (19), and (22) we know that $\frac{\partial A}{\partial p} < 0$, $\frac{\partial \mathcal{A}}{\partial p} < 0$, $\frac{\partial \tilde{w}_1}{\partial p} < 0$ and $\frac{\partial \tilde{w}_0}{\partial p} \geq 0$, which means an increase in the unit price of tax avoidance schemes will increase the minimum investment requirement of fettered taxpayers, $A_i = \tilde{w}_0$, and decrease the threshold of avoidance of unfettered taxpayers, $\frac{\tau}{p}$, and hence the critical value of wealth of unfettered taxpayers, $\tilde{w}_1$. However, the reduction in avoidance of fettered taxpayers induced by the increase of the minimum investment requirement is greater than the increase in avoidance of unfettered taxpayers induced by the decrease the threshold of avoidance of unfettered taxpayers. Therefore, the aggregate avoidance A in the market decrease. So we have Proposition 4.

Proposition 4 *The extensive margin of fettered taxpayers is greater than the intensive margin of unfettered taxpayers.*

In summary, in equilibrium, the aggregate avoidance A becomes

$$A = \int A(w)\, g(w)\ \mathrm{d}w = \int_{\tilde{w}_0}^{\infty} w g(w)\ \mathrm{d}w = \mu_{w \geqslant \tilde{w}_0} \left[1 - G(\tilde{w}_0)\right], \tag{28}$$

where $\mu_{w \geqslant \tilde{w}_0} = \frac{\int_{\tilde{w}_0}^{\infty} w g(w)\ \mathrm{d}w}{1 - G(\tilde{w}_0)}$.

In equilibrium, both fettered and unfettered taxpayers avoid all their wealth which means the tax authority could only collect tax from those low-income taxpayers who are excluded from the tax avoidance market by the minimum investment requirement. Therefore, the expected tax revenue of the tax authority is

$$R = \int \left[\rho t w_i + (1-\rho)\, t\, (w_i - A_i)\right] g(w)\ \mathrm{d}w \tag{29}$$

$$= \rho t \int_0^{\tilde{w}_0(t)} w g(w)\ \mathrm{d}w \tag{30}$$

And, the expected profit of the firm is given by

$$\begin{aligned} E(\pi) &= \frac{p^*}{N} \int_{\tilde{w}_1}^{\infty} \mathcal{A} w g(w)\ \mathrm{d}w - \frac{\tau}{N} \int_{\tilde{w}_1}^{\infty} g(w)\ \mathrm{d}w - c_L \rho_L - \upsilon \\ &= \left(p^* \mu_{w > \tilde{w}_1} - \tau\right) \left[1 - G(\tilde{w}_1)\right] - c_L \rho_L - \upsilon. \end{aligned} \tag{31}$$

where $\tilde{w}_1 = \frac{\tau}{p^*}$ and $\mu_{w > \tilde{w}_1} = \frac{\int_{\tilde{w}_1}^{\infty} w g(w)\ \mathrm{d}w}{1 - G(\tilde{w}_1)}$.

4 Simulation

To make further progress, we assess the properties of optimal avoidance via a numerical optimization procedure. First, we simulate market structure of tax avoidance. By reading the National Audit Office report, we know that (1) there are currently between 50 and 100 active promoters in the market; (2) HMRC estimates there are 30,000 users of partnership loss schemes and employment intermediary schemes; (3) 110 cases entered litigation from April 2010 to October 2012 and 60 cases of them were judged and HMRC was successful in 51, so the successful rate is around 0.85 (The Comptroller and Auditor General, 2012). H.M. Revenue and Customs has won a legal case over tax avoidance scheme promoter Hyrax Resourcing Ltd, which will help the tax authority collect over £40 million in unpaid taxes. The scheme promoted by Hyrax was a disguised remuneration avoidance scheme which worked by paying scheme users in loans so they could avoid paying Income Tax and National Insurance on their earnings. Hyrax Resourcing Limited accepted applications from users, created employment contracts, signed service contracts, paid employees and transferred loan agreements to offshore trusts. Scheme users were paid just enough to comply with the National Minimum Wage. The rest of their income was made up in loans which were transferred to an offshore trust in Jersey. The amounts received under loan agreements were not declared as income on the scheme users tax return, meaning they didn't pay tax on all their earnings. Scheme users paid Hyrax 18% promoter fees to allow them to access the scheme (H.M. Revenue and Customs and The Rt Hon Mel Stride MP, 2019). Therefore, we use 18% as the equilibrium price of our baseline model. The average disposable income in the UK is £34,210 and the median is £28,418 in 2017-2018 (Office for National Statistics). We model the UK income distribution as lognormal. Using the published mean and median of the UK income distribution, we estimate that μ and σ (mean and variance parameters) are equal to 10.2548 and 0.60909 of the lognormal.

Figure 2 depicts the relationship between tax rate and cut-off point of wealth for avoidance. We can see that $\tilde{w}_0$ is decreasing in tax rate when it is lower than 0.5 and at the beginning, $\tilde{w}_0$ is shrinking quickly and then slowly. Figure 3 depicts the relationship between tax rates and aggregate avoidance. The intensive margin is zero (unfettered taxpayers continue to avoid all their wealth), so there are three effects that determine aggregate avoidance: negative income effect, substitution effect and extensive margin effect. Negative income effect means the increase in tax rate makes taxpayer poorer. Poorer taxpayer becomes more risk averse (because log utility implies decreasing absolute risk aversion). So more risk averse taxpayer wishes to decrease avoidance. Substitution effect means avoidance becomes more valuable as the tax rate is higher. So increase in tax rate makes the taxpayer want to avoid more. When tax rate is lower than 0.5, substitution effect dominates and aggregate avoidance is increasing; when tax rate is higher than 0.5, income effect dominates and aggregate avoidance is deceasing. Extensive margin effect is that increasing tax rate leads to $\tilde{w}_0$ fall, which means more avoiders, the fettered taxpayers enter the avoidance market and so aggregate avoidance increase. Figure 2 explains why aggregate avoidance start to grow fast and then slowly in Figure 3. Figure 4 depicts the relationship between tax rate and tax revenue, which is actually the Laffer curve when tax rate is less than 0.5. There is one more effect - intensive margin effect. Even the aggregate avoidance is still the same (unfettered taxpayers continue to avoid all their wealth), increasing tax rate makes the existing avoiders avoid more tax. Increasing tax rate raises the tax revenue from non-avoiders, but also significantly reduces tax revenue from more fettered taxpayers (extensive margin effect) and the unfettered avoiding more tax (intensive margin effect). So there is a sharp decrease in tax revenue when substitution effect dominates, which shapes the Laffer curve. There is a trade-off between tax revenue and aggregate avoidance and how to balance them depends on the government's goal. When tax rate is lower than the revenue maximizing tax rate, an increase in tax rate increases both aggregate avoidance and total tax revenue. When tax rate is higher than the revenue maximizing tax rate, increasing tax rate will not only reduce tax revenue but also increase aggregate avoidance.

Other numerical generated results we have analysed—which we don not report here for brevity—indicate that the qualitative nature of the results given in related comparative statics continue to hold.

5 Conclusion

Tax avoidance is estimated to cost the UK government £1.8 billion of income tax revenues from 2017 to 2018. Previous studies only discuss the demand side of tax avoidance but we recognise the existence of a competitive 'tax advice' industry supplying tax avoidance schemes which help taxpayers reduce their tax liability. We start from the demand of tax avoidance of taxpayers and combine the supply side of the tax avoidance market to provide an analysis which address the abuse of marketed tax avoidance schemes. It is assumed that there is a continuum of risk-averse taxpayers buying such schemes from a tax advice firm at per unit market price, subject to a minimum investment induced by the existence of the customer-specific set-up cost. The tax authority may mount a legal challenge to the scheme and, if successful, it can only reclaim the tax owed but cannot levy a fine. The firm faces a legal cost if schemes are challenged by the tax authority and has other fixed costs. Under these assumptions, we find that (1) the individual's demand for avoidance is a function of wealth: fettered may choose to avoid an amount $\frac{\tau}{p}$ or all his wealth and unfettered taxpayers avoid the optimal proportion of their income $\mathcal{A}$ at the per unit market price; (2) the unit price of tax avoidance schemes is affected by the tax rate, the probability of successful legal challenge, and the proportion of avoided income to total income; (3) tax avoidance of unfettered taxpayers is price elastic, so firms impose no upper limits on the amount that can be avoided; (4) so in equilibrium, both fettered and unfettered taxpayers avoid all their wealth, but fettered taxpayers bear a higher per unit price to avoid tax than unfettered taxpayers in general; (5) the extensive margin of fettered taxpayers is greater than the intensive margin of unfettered taxpayers; (6) by simulation, we find the Laffer curve indicating the tax authority should make appropriate tax rate if the goal is maximising tax revenue.

Our model provides a rich framework for understanding how the supply side of the tax avoidance market affects taxpayers' avoidance behavior. However, in our model, taxpayers are only allowed to buy avoidance schemes at one firm while in reality, some taxpayers will buy different classes of avoidance schemes at different firms to spread avoided income so that they can mitigate audit or legal challenge risks. We leave it for future research.

Appendix

Proof of proposition 1. Rearranging 18, we obtain and define

$$F(\tilde{w}_0) = U[w^s(\tilde{w}_0, 0)] - \rho U[w^s(\tilde{w}_0, \tau)] - (1-\rho) U[w^u(\tilde{w}_0, A_i, \tau)]. \tag{A.1}$$

If $A_i = \frac{\tau}{p}$, let

$$F(\tilde{w}_0') = U(w^s(0, \tilde{w}_0')) - \rho U[w^s(\tau, \tilde{w}_0')] - (1-\rho) U\left[w^u\left(\tau, \frac{\tau}{p}, \tilde{w}_0'\right)\right] \tag{A.2}$$

Then

$$F(\tilde{w}_0') = 0$$

If $A_i = w_i$, let

$$F(\tilde{w}_0'') = U(w^s(0, \tilde{w}_0'')) - \rho U[w^s(\tau, \tilde{w}_0'')] - (1-\rho) U[w^u(\tau, \tilde{w}_0'', \tilde{w}_0'')] \tag{A.3}$$

Then

$$F\left(\tilde{w}_0''\right) = 0$$

So

$$\tilde{w}_0 = \begin{cases} \tilde{w}_0' & \text{if } \tau < p\tilde{w}_0' \\ \tilde{w}_0'' & \text{otherwise} \end{cases}$$

From equation 14 to A.3, we get the piecewise functions of avoided income $A_i \equiv A\left(w, \tilde{w}_0\right)$. For $\tilde{w}_0 = \tilde{w}_0^{'}$,

$$A_i \equiv A\left(w, \tilde{w}_0^{'}\right) = \begin{cases} \mathcal{A}w_i & \text{if } w_i > \tilde{w}_1 \\ \frac{\tau}{p} & \text{if } w_i \in [\tilde{w}_0', \tilde{w}_1] \\ 0 & \text{otherwise} \end{cases} \tag{A.4}$$

The A.4 lists the piecewise functions of avoided income on wealth. It shows that the taxpayer avoids with only one firm j and so he or she puts all avoided income in that firm. When his or her wealth is more than $\tilde{w}_1$ he or she will choose to avoid a part $\mathcal{A}$ of his wealth (the optimal avoided income A_i^*) with a firm from N firms at random; when his or her wealth is between $\tilde{w}_0'$ and $\tilde{w}_1$ he or she could only choose to avoid an amount $A_i = \frac{\tau}{p}$ with a firm from N firms at random; otherwise, the taxpayer will not avoid and so the avoided income is 0.

In this case, the aggregate avoidance is given by

$$A\mid_{\tilde{w}_0=\tilde{w}_0^{'}} = \int_{\tilde{w}_0'}^{\tilde{w}_1} \frac{\tau}{p} g(w)\ \mathrm{d}w + \int_{\tilde{w}_1}^{\infty} \mathcal{A}wg(w)\ \mathrm{d}w \tag{A.5}$$

For $\tilde{w}_0 = \tilde{w}_0^{''}$, the piecewise functions of avoided income is given by Proposition 1.

In this case, the aggregate avoidance is given by

$$A\mid_{\tilde{w}_0=\tilde{w}_0^{''}} = \int_{\tilde{w}_0''}^{\frac{\tau}{p}} wg(w)\ \mathrm{d}w + \int_{\frac{\tau}{p}}^{\tilde{w}_1} \frac{\tau}{p} g(w)\ \mathrm{d}w + \int_{\tilde{w}_1}^{\infty} \mathcal{A}wg(w)\ \mathrm{d}w \tag{A.6}$$

■

Proof of Lemma 1. Since $g\left(0\right) = t(1-t)(1-\rho) > 0$ and $g\left(t\right) = -\rho t(1-t) < 0$, there are odd number of roots in the interval $(0, t)$. We know that quadratic function have maximum two roots, therefore, there is only one root in the interval $\left(0, t\right)$. Solving equation 10 for p

we get the inverse demand function. We begin by proving $p > 0$. From equation (11)

$$
\begin{aligned}
p &> 0 \Leftrightarrow t > \frac{\mathcal{A}t-(1-t)+\sqrt{\left[\mathcal{A}t-(1-t)\right]^2+4\mathcal{A}\rho(1-t)t}}{2\mathcal{A}} && \text{(A.7)}\\
&\Leftrightarrow \mathcal{A}t+1-t > \sqrt{\left[\mathcal{A}t-(1-t)\right]^2+4\mathcal{A}\rho(1-t)t} && \text{(A.8)}\\
&\Leftrightarrow (\mathcal{A}t+1-t)^2 > \left[\mathcal{A}t-(1-t)\right]^2+4\mathcal{A}\rho(1-t)t && \text{(A.9)}\\
&\Leftrightarrow 4\mathcal{A}(1+\rho)(1-t)t > 0 && \text{(A.10)}
\end{aligned}
$$

Since $4\mathcal{A}(1+\rho)(1-t)t > 0$ holds, this is consistent with the proof.

Then we prove $p < t$.

$4\mathcal{A}\rho(1-t)t > 0$, so $\sqrt{(\mathcal{A}t-(1-t))^2+4\mathcal{A}\rho(1-t)t} > \mathcal{A}t-(1-t)$ and the second term of equation 11 is greater than zero no matter what is the sign of $\mathcal{A}t-(1-t)$. Therefore, $p < t$ holds where $t \in (0,1)$. ■

Proof of Proposition 2. Differentiating equation 8 with respect to p gives

$$\frac{\partial\mathcal{A}}{\partial p} = \frac{2\mathcal{A}p-\mathcal{A}t-(1-t)}{p\,(t-p)} < 0$$

So the inverse price elasticity of optimal proportion of tax avoidance to wealth is given by

$$\varepsilon_{p,\mathcal{A}} = -\frac{\mathcal{A}}{p}\frac{\partial p}{\partial \mathcal{A}} = \frac{1}{2} + \frac{\mathcal{A}t-(1-t)}{2\sqrt{\left[\mathcal{A}t-(1-t)\right]^2+4\mathcal{A}\rho(1-t)t}} \tag{A.11}$$

From equation (A.11) we note that $-\frac{1}{2} = \frac{-\sqrt{\left[\mathcal{A}t-(1-t)\right]^2+4\mathcal{A}\rho(1-t)t}}{2\sqrt{\left[\mathcal{A}t-(1-t)\right]^2+4\mathcal{A}\rho(1-t)t}} < \frac{\mathcal{A}t-(1-t)}{2\sqrt{\left[\mathcal{A}t-(1-t)\right]^2+4\mathcal{A}\rho(1-t)t}} < \frac{\sqrt{\left[\mathcal{A}t-(1-t)\right]^2+4\mathcal{A}\rho(1-t)t}}{2\sqrt{\left[\mathcal{A}t-(1-t)\right]^2+4\mathcal{A}\rho(1-t)t}} = \frac{1}{2}$ holds irrespective of the sign of $\mathcal{A}t-(1-t)$, after plus $\frac{1}{2}$, so the inverse price elasticity of optimal proportion of tax avoidance to wealth is $\varepsilon_{p,\mathcal{A}} \in (0,1)$, which means $\varepsilon_{\mathcal{A},p} = \frac{1}{\varepsilon_{p,\mathcal{A}}} > 1$.

We define that A_u is total tax avoidance of unfettered taxpayers

$$A_u = \int_{\tilde{w}_1}^{\infty} \mathcal{A}wg(w)\ \mathrm{d}w = \mathcal{A}\mu_{w>\tilde{w}_1}\left[1-G\left(\tilde{w}_1\right)\right] \tag{A.12}$$

Differentiating A_u with respect to p we get

$$\frac{\partial A_u}{\partial p} = \frac{\partial \mathcal{A}}{\partial p}\int_{\tilde{w}_1}^{\infty} wg(w)\ \mathrm{d}w - \mathcal{A}\tilde{w}_1 g(\tilde{w}_1)\frac{\partial \tilde{w}_1}{\partial p} < 0 \tag{A.13}$$

So we get the price elasticity of tax avoidance of unfettered taxpayers

$$\varepsilon_{A_u,p} = -\frac{p}{A_u}\frac{\partial A_u}{\partial p} = -\frac{p}{A_u}\left[\frac{\partial \mathcal{A}}{\partial p}\int_{\tilde{w}_1}^{\infty} wg(w)\ \mathrm{d}w - \mathcal{A}\tilde{w}_1 g(\tilde{w}_1)\frac{\partial \tilde{w}_1}{\partial p}\right] \tag{A.14}$$

From equation 14 we know ($\tilde{w}_1$ is a function of p)

$$\frac{\partial \tilde{w}_1}{\partial p} = \frac{\rho t \tau}{(1-t)(t-p-\rho t)^2} > 0$$

Rearranging equation A.14, we get the price elasticity of tax avoidance of unfettered taxpayers

$$\varepsilon_{A_u,p} = \frac{A_u \varepsilon_{\mathcal{A},p} + \tau g(\tilde{w}_1)\frac{\partial \tilde{w}_1}{\partial p}}{A_u}$$

We know $\frac{\partial \tilde{w}_1}{\partial p} > 0$ and $\varepsilon_{\mathcal{A},p} > 1$, so $\varepsilon_{A_u,p} > 1$ which means the price elasticity of tax avoidance of unfettered taxpayers is elastic. ■

Proof of Proposition 3. For fettered taxpayers, they can only choose to avoid an amount $A_i = \frac{\tau}{p}$ or their total income $A_i = w_i$. So we need check whether this two cases hold in equilibrium. By contradiction we can prove that, in equilibrium, $\tilde{w}_0 = \tilde{w}_0^{'}$ is invalid and $\tilde{w}_0 = \tilde{w}_0^{''}$ holds.

In equilibrium ($A_i = w_i$), setting $\tilde{w}_0 = \frac{\tau}{p}$ and from equation 18 we obtain

$$\begin{aligned}
U\left(w^s\left(0,\tilde{w}_0\right)\right) &= \rho U\left(w^s\left(\tau,\tilde{w}_0\right)\right) + (1-\rho)\,U\left(w^u\left(\tau,A_i,\tilde{w}_0\right)\right) \\
U\left[(1-t)\,\tilde{w}_0\right] &= \rho U\left[(1-t)\,\tilde{w}_0 - p\tilde{w}_0\right] + (1-\rho)\,U\left[(1-t)\,\tilde{w}_0 + t\tilde{w}_0 - p\tilde{w}_0\right] && \text{(A.15)} \\
U\left[(1-t)\,\tilde{w}_0\right] &= \rho U\left[(1-t-p)\,\tilde{w}_0\right] + (1-\rho)\,U\left[(1-p)\,\tilde{w}_0\right] && \text{(A.16)} \\
U\left(1-t\right) &= \rho U\left(1-t-p\right) + (1-\rho)\,U\left(1-p\right) && \text{(A.17)}
\end{aligned}$$

Given that $p = p^*(\rho)$ the .A.17 are $\rho = 0$ (in which case $p^*(0) = t$) and $\rho = 1$ (in which case $p^*(1) = 0$). This is the contradiction immediately, since ρ cannot be either 0 or 1 at an interior optimum for the fettered taxpayer. In other words, $\tilde{w}_0 = \frac{\tau}{p}$ does not holds. Therefore, $\tilde{w}_0 < \frac{\tau}{p}$ holds given $\tilde{w}_0 < \tilde{w}_1 = \frac{\tau}{p}$.

Given our assumption, taxpayers could not avoid more than their wealth, so the case $\tilde{w}_0 = \tilde{w}_0^{'}$ implicitly indicates $\tilde{w}_0^{'} > \frac{\tau}{p}$. This is the contradiction immediately, therefore, $\tilde{w}_0 = \tilde{w}_0^{'}$ is invalid and $\tilde{w}_0 = \tilde{w}_0^{''}$ holds in equilibrium. ■

Summary of Comparative Statics. When $\tilde{w}_0 = \tilde{w}_0^{'}$, it holds for A that

$$\frac{\partial A}{\partial p} = \frac{\partial \mathcal{A}}{\partial p}\left[1-G(\tilde{w}_1)\right]\mu_{w\geq\tilde{w}_1} - \frac{\tau}{p}g(\tilde{w}_0^{'})\frac{\partial \tilde{w}_0^{'}}{\partial p} - \frac{\tau}{p^2}\left[G(\tilde{w}_1)-G(\tilde{w}_0^{'})\right] < 0 \tag{A.18}$$

$$\frac{\partial A}{\partial \rho} = \frac{\partial \mathcal{A}}{\partial \rho}\left[1-G(\tilde{w}_1)\right]\mu_{w\geq\tilde{w}_1} - \frac{\tau}{p}g(\tilde{w}_0^{'})\frac{\partial \tilde{w}_0^{'}}{\partial \rho} < 0 \tag{A.19}$$

$$\frac{\partial A}{\partial t} = \frac{\partial \mathcal{A}}{\partial t}\left[1-G(\tilde{w}_1)\right]\mu_{w\geq\tilde{w}_1} - \frac{\tau}{p}g(\tilde{w}_0^{'})\frac{\partial \tilde{w}_0^{'}}{\partial t} \gtrless 0 \tag{A.20}$$

$$\frac{\partial A}{\partial \tau} = \frac{1}{p}\left[G(\tilde{w}_1)-G(\tilde{w}_0^{'})-\tilde{w}_0^{'}g(\tilde{w}_0^{'})\right] \gtrless 0 \tag{A.21}$$

When $\tilde{w}_0 = \tilde{w}_0^{''}$, it holds for A that

$$\frac{\partial A}{\partial p} = \frac{\partial \mathcal{A}}{\partial p}\left[1-G(\tilde{w}_1)\right]\mu_{w\geq\tilde{w}_1} - \frac{\tau}{p^2}\left[G(\tilde{w}_1)-G(\frac{\tau}{p})\right] < 0 \tag{A.22}$$

$$\frac{\partial A}{\partial \rho} = \frac{\partial \mathcal{A}}{\partial \rho}\left[1-G(\tilde{w}_1)\right]\mu_{w\geq\tilde{w}_1} - \tilde{w}_0^{''}g(\tilde{w}_0^{''})\frac{\partial \tilde{w}_0^{''}}{\partial \rho} < 0 \tag{A.23}$$

$$\frac{\partial A}{\partial t} = \frac{\partial \mathcal{A}}{\partial t}\left[1-G(\tilde{w}_1)\right]\mu_{w\geq\tilde{w}_1} - \tilde{w}_0^{''}g(\tilde{w}_0^{''})\frac{\partial \tilde{w}_0^{''}}{\partial t} \gtrless 0 \tag{A.24}$$

$$\frac{\partial A}{\partial \tau} = \frac{1}{p}\left[G(\tilde{w}_1)-G(\frac{\tau}{p})\right] - \tilde{w}_0^{''}g(\tilde{w}_0^{''})\frac{\partial \tilde{w}_0^{''}}{\partial \tau} \gtrless 0 \tag{A.25}$$

Equation A.2 implies the second derivative of F' with respect to $\tilde{w}_0^{'}$ which is given by

$$d' = \frac{\partial F'}{\partial \tilde{w}_0^{'}} - \frac{\mathcal{A}\tau(t-p)\left(\tilde{w}_1-\tilde{w}_0^{'}\right)}{\tilde{w}_0^{'}w^s\left(\tau,\tilde{w}_0^{'}\right)w^u\left(\tau,\frac{\tau}{p},\tilde{w}_0^{'}\right)} < 0 \tag{A.26}$$

Differentiating equation A.2 we obtain

$$\frac{\partial \tilde{w}_0^{'}}{\partial p} = -\frac{\partial F'/\partial p}{\partial F'/\partial \tilde{w}_0^{'}} = -\frac{(1-\rho)t\tau}{p^2 d' w^u\left(\tau,\frac{\tau}{p},\tilde{w}_0^{'}\right)} > 0 \tag{A.27}$$

$$\frac{\partial \tilde{w}_0^{'}}{\partial \rho} = -\frac{\log\left[w^u\left(\tau,\frac{\tau}{p},\tilde{w}_0^{'}\right)\right]-\log\left[w^s\left(\tau,\tilde{w}_0^{'}\right)\right]}{d'} > 0 \tag{A.28}$$

$$\frac{\partial \tilde{w}_0^{'}}{\partial \tau} = \frac{\tilde{w}_0^{'}}{\tau} > 0 \tag{A.29}$$

$$\frac{\partial \tilde{w}_0^{'}}{\partial t} = \frac{1}{d'}\left\{\frac{(1-\rho)\left(\frac{\tau}{p}-\tilde{w}_0^{'}\right)}{(1-t)\tilde{w}_0^{'}-\tau+\frac{t\tau}{p}} - \frac{\rho\tilde{w}_0^{'}}{(1-t)\tilde{w}_0^{'}-\tau} + \frac{1}{1-t}\right\} \gtrless 0 \tag{A.30}$$

Equation A.3 implies the second derivative of F with respect to $\tilde{w}_0$ which is given by

$$d'' = \frac{\partial F''}{\partial \tilde{w}_0^{''}} = -\frac{\tau\left[w^s\left(\tau,\tilde{w}_0^{''}\right)+\rho t\tilde{w}_0^{''}\right]}{\tilde{w}_0^{''}w^u\left(\tau,\tilde{w}_0^{''},\tilde{w}_0^{''}\right)w^s\left(\tau,\tilde{w}_0^{''}\right)} < 0 \tag{A.31}$$

Differentiating equation A.3 we obtain

$$\frac{\partial \tilde{w}_0''}{\partial p} = 0 \tag{A.32}$$

$$\frac{\partial \tilde{w}_0''}{\partial \rho} = -\frac{\log\left[w^u\left(\tau, \tilde{w}_0'', \tilde{w}_0''\right)\right] - \log\left[w^s\left(\tau, \tilde{w}_0''\right)\right]}{d''} > 0 \tag{A.33}$$

$$\frac{\partial \tilde{w}_0''}{\partial \tau} = -\frac{w^s\left(\tau, \tilde{w}_0''\right) + \rho t \tilde{w}_0''}{w^u\left(\tau, \tilde{w}_0'', \tilde{w}_0''\right) w^s\left(\tau, \tilde{w}_0''\right) d''} > 0 \tag{A.34}$$

$$\frac{\partial \tilde{w}_0''}{\partial t} = \frac{(1-\rho)(1-t)\tilde{w}_0'' - \tau}{d''(1-t) w^s\left(\tau, \tilde{w}_0''\right)} \gtrless 0 \tag{A.35}$$

■

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List of Figures

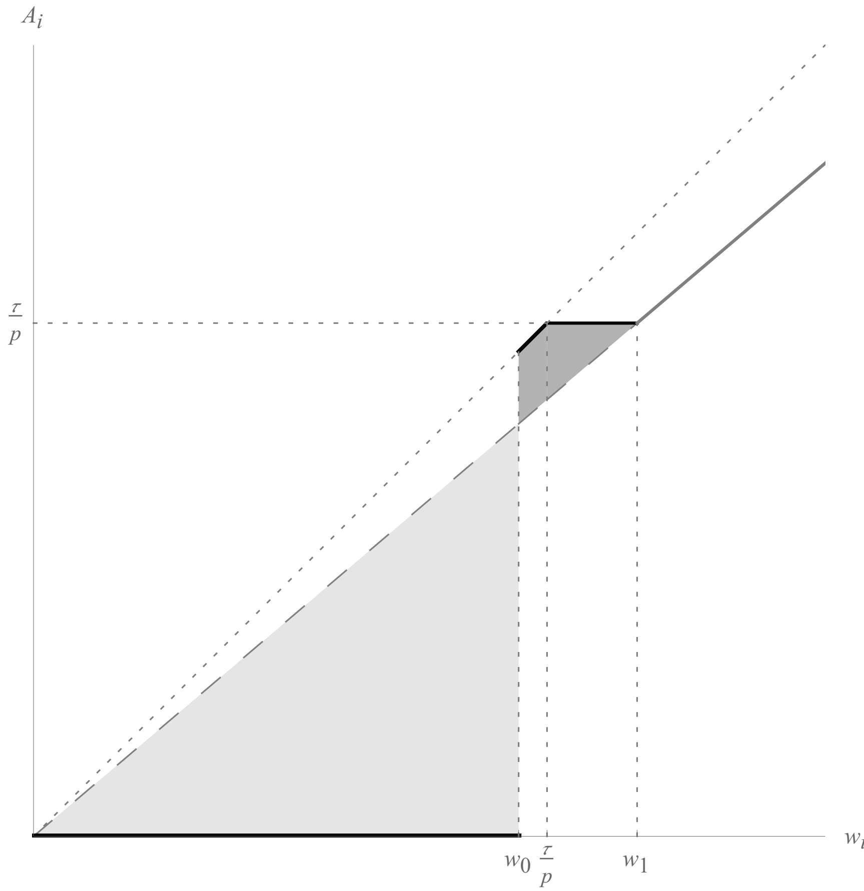

Figure 1: Avoidance Behaviour

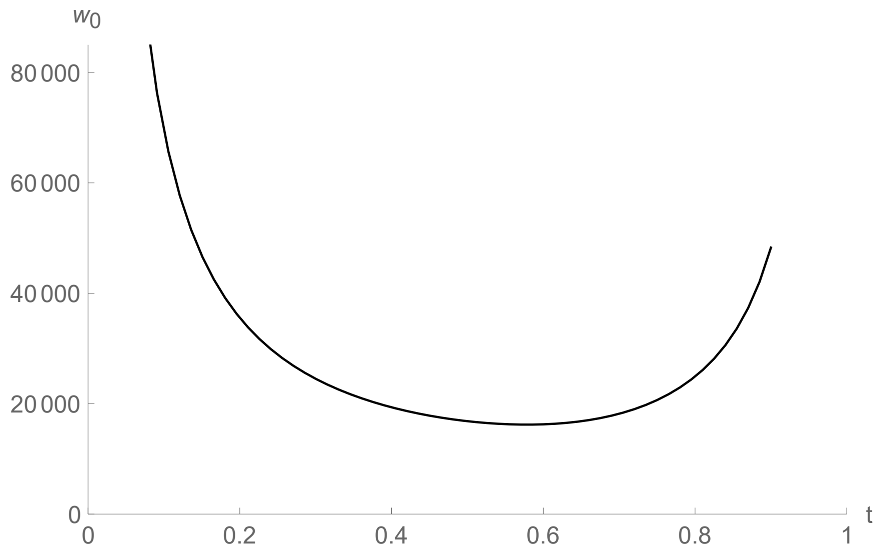

Figure 2: The relationship between tax rate and cut-off point of wealth for avoidance

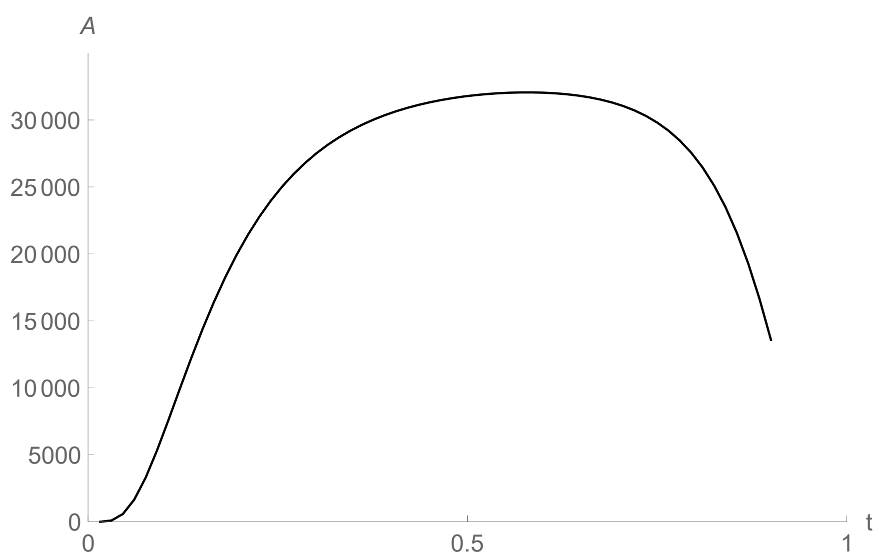

Figure 3: The relationship between tax rate and aggregate avoidance

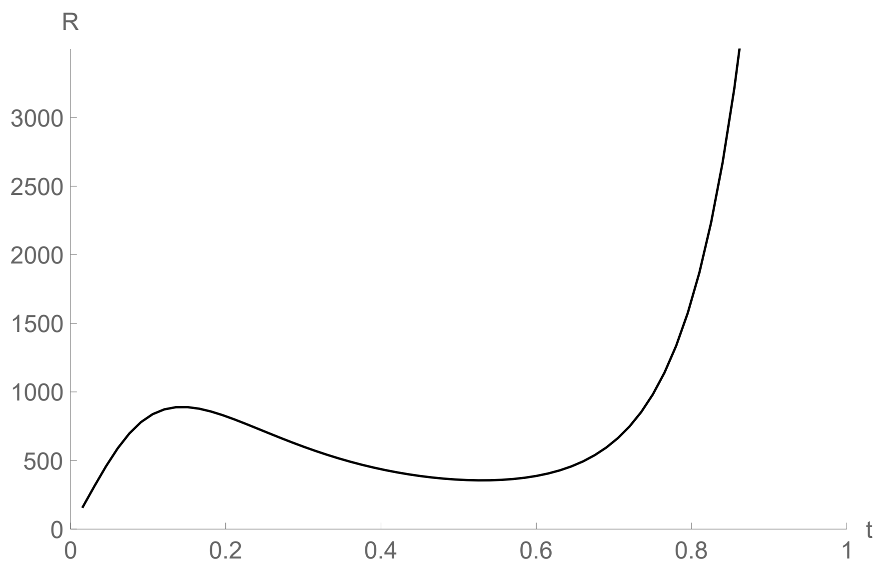

Figure 4: The relationship between tax rate and tax revenue